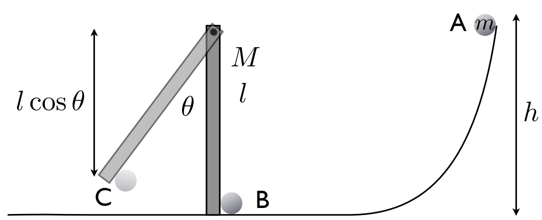
Problem Set 5

Instructions:

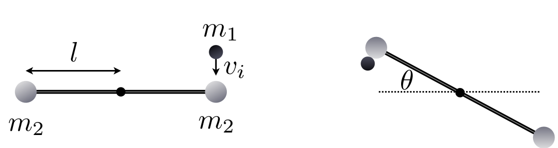
1. Answer all questions below. Show your work for full credit.
2. All problems are due by the end of the day on 21 March 2014
3. Late penalties will not be incurred until after spring break
4. You may collaborate, but everyone must turn in their own work.

1. A bowler throws a bowling ball of radius R along a lane. The ball slides on the lane with initial speed v_o and initial angular speed $\omega_o = 0$. The coefficient of kinetic friction between the ball and the lane is μ_k. The kinetic frictional force $\vec{\mathbf{f}}_k$ acting on the ball causes a linear acceleration of the ball while producing a torque that causes an angular acceleration of the ball. When the center of mass speed v_{cm} has decreased enough and angular speed ω has increased enough, the ball stops sliding and then rolls smoothly. **(a)** What then is the center of mass speed v_{cm} in terms of ω? During the sliding, what are the ball's **(b)** linear acceleration and **(c)** angular acceleration? **(d)** How long does the ball slide? **(e)** How far does the ball slide? **(f)** What is the linear speed of the ball when smooth rolling begins?

2. In the figure below, a small block of mass m slides down a frictionless surface through height h and then sticks to a uniform rod of mass M and length L. The rod pivots about point O through angle θ before momentarily stopping. Find θ.



3. In the figure below, two balls of mass m are attached to the ends of a thin rod of length L and negligible mass. The rod is free to rotate in a vertical plane without friction about a horizontal axis through its center. With the rod initially horizontal, a wad of wet clay of mass M drops onto one of the balls, hitting it with a speed of v_i and then sticking to it. **(a)** What is the angular speed of the system just after the putty wad hits? **(b)** What is the ratio of the kinetic energy of the system after the collision to that of the putty wad just before? **(c)** Through what angle will the system rotate before it momentarily stops?



4. Archimedes supposedly was asked to determine whether a crown made for the king consisted of pure gold. Legend has it that he solved this problem by weighing the crown first in air and then in water. Suppose the scale read 7.84 N in air and 6.84 N while submersed in water. What should Archimedes have told the king? (Note: $\rho_{\text{water}} = 1000\,\text{kg/m}^3$, $\rho_{\text{gold}} = 19.3 \times 10^3\,\text{kg/m}^3$

5. Cylindrical pressure vessels are often reinforced by braided fibers, wound at a specific angle. For a thin-walled pipe, two times of forces are crucial to the overall strength of the pipe: the forces trying to expand the pipe in the radial direction, and those trying to elongate the pipe (in engineering parlance, we are talking about hoop stresses and axial stresses[i]). **(a)** What is the relationship between the radial and axial forces, per unit area? **(b)** The winding reinforces the pipe most effectively when the axial and radial components of the tension in the fiber have the same ratio as the axial and radial forces. Given that constraint, what is the optimal winding angle α?

6. The density of water is $1000\,\text{kg/m}^3$, while that of ice is $916.7\,\text{kg/m}^3$. If a block of ice is placed in water, what volume fraction of the ice is below the surface?

7. Superman attempts to drink water through a very long straw. With his great strength, he achieves maximum possible suction. The walls of the straw do not collapse. Find the maximum height through which he can lift the water.

8. Viscosity of most liquids can be represented by an extra "drag" force on a body moving in a liquid, which is reasonably well approximated by $F_{\text{drag}} = 6\pi\eta R v$, where v is the velocity of the body, η is the viscosity parameter of the fluid, and R is a characteristic dimension of the falling object (the radius, in the case of a sphere). The presence of viscosity leads to a "terminal velocity" of a body falling in a liquid.

Consider a sphere of radius R and density ρ_s falling through a liquid of density ρ_l and viscosity parameter η. Including this new drag force, the buoyant force, and the weight of the object, find an expression for the terminal velocity of the sphere.

[i] Stress is just a generalization of pressure for solid objects, it is a force per unit area just like pressure. You may find the following useful: `http://en.wikipedia.org/wiki/Cylinder_stress`.