Problem Set 6

Instructions:

1. Answer all questions below. Show your work for full credit.
2. All problems are due by the end of the day on 14 April 2014
3. Late penalties will not be incurred until after spring break
4. You may collaborate, but everyone must turn in their own work.

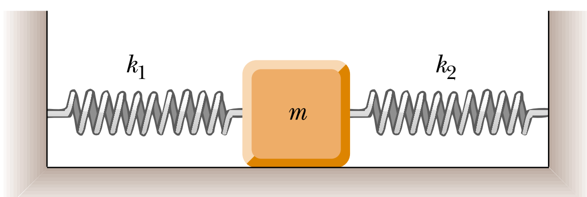
1. An object of mass m is dropped from a height h above the surface of a planet of mass M and radius R. Assume the planet has no atmosphere so that friction can be ignored.

(a) What is the speed of the mass just before it strikes the surface of the planet? Do *not* assume that h is small compared with R.

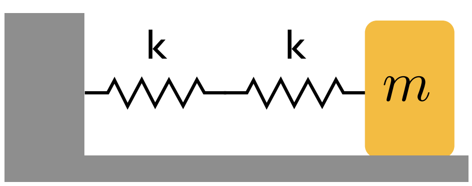
(b) Show that the expression from **(a)** reduces to $v = \sqrt{2gh}$ for $h \ll R$.

(c) How long does it take for the object to fall to the surface for an arbitrary value of h? Use any means necessary to evaluate the integral required. Bonus points (10%) for code submissions.

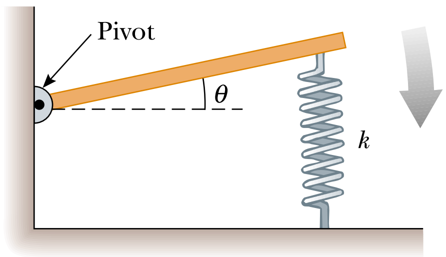
2. A block of mass m is connected to two springs of force constants k_1 and k_2 as shown below. The block moves on a frictionless table after it is displaced from equilibrium and released. Determine the period of simple harmonic motion. (Hint: what is the total force on the block if it is displaced by an amount x?



3. A mass m is connected to two springs in series as shown below. What is the period of simple harmonic motion if the mass is displaced from equilibrium. *Hint: what must be true of the displacement of each spring if the total displacement is Δx?*



4. A horizontal plank of mass m and length L is pivoted at one end. The plank's other end is supported by a spring of force constant k. The moment of inertia of the plank about the pivot is $I = \frac{1}{3}mL^2$. The plank is displaced by a small angle θ from horizontal equilibrium and released. Find the angular frequency ω of simple harmonic motion. (Hint: consider the torques about the pivot point.)



5. Assume the earth to be a solid sphere of uniform density. A hole is drilled through the earth, passing through its center, and a ball is dropped into the hole. Neglect friction.

(a) Calculate the time for the ball to return to the release point. *Hint:* what sort of motion results?
(b) Compare the result of part a to the time required for the ball to complete a circular orbit of radius R_E about the earth

6. A satellite is in a circular Earth orbit of radius r. The area A enclosed by the orbit depends on r^2 because $A = \pi r^2$. Determine how the following properties depend on r: **(a)** period, **(b)** kinetic energy, **(c)** angular momentum, and **(d)** speed.

7. Here are some functions:

$$f_1(x,t) = Ae^{-b(x-vt)^2} \qquad f_2(x,t) = \frac{A}{b\,(x-vt)^2+1}$$

$$f_3(x,t) = Ae^{-b\left(bx^2+vt\right)} \qquad f_4(x,t) = A\sin(bx)\cos(bvt)^3$$

(a) Which ones satisfy the wave equation? Justify your answer with explicit calculations.
(b) For those functions that *satisfy* the wave equation, write down the corresponding functions $g(x,t)$ representing a wave of the same shape traveling in the opposite direction.

8. The equation for a driven damped oscillator is

$$\frac{d^2x}{dt^2} + 2\gamma\omega_o\frac{dx}{dt} + \omega_o^2 x = \frac{q}{m}E(t) \tag{1}$$

(a) Explain the significance of each term.
(b) Let $E = E_o e^{i\omega t}$ and $x = x_o e^{i(\omega t-\alpha)}$ where E_o and x_o are real quantities. Substitute into the above expression and show that

$$x_o = \frac{qE_o/m}{\sqrt{\left(\omega_o^2-\omega^2\right)^2+\left(2\gamma\omega\omega_o\right)^2}} \tag{2}$$

(c) Derive an expression for the phase lag α, and sketch it as a function of ω, indicating ω_o on the sketch.

9. **(a)** A diatomic molecule has only one mode of vibration, and we may treat it as a pair of masses connected by a spring (figure (a) below). Find the vibrational frequency, assuming that the masses of A and B are different. Call them m_a and m_b, and let the spring have constant k.

(b) A diatomic molecule adsorbed on a solid surface (figure (b) below) has more possible modes of vibration. Presuming the two springs and masses to be equivalent this time, find their frequencies.

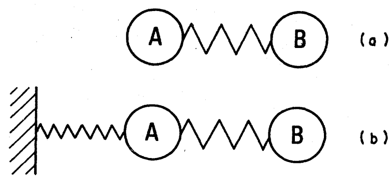


FIG. 1. (a) Classical model for vibrating free-space AB diatomic molecule; (b) same molecule adsorbed onto a surface.

Figure 1: *From http://prb.aps.org/abstract/PRB/v19/i10/p5355_1.*

10. *Energetics of diatomic systems* An approximate expression for the potential energy of two ions as a function of their separation is

$$PE = -\frac{ke^2}{r} + \frac{b}{r^9} \tag{3}$$

The first term is the Coulomb interaction representing the electrical attraction of the two ions, while the second term is introduced to account for the repulsive effect of the two ions at small distances. **(a)** Find b as a function of the equilibrium spacing r_o. (What must be true at equilibrium?) **(b)** For KCl, with an equilibrium spacing of $r_o = 0.279\,\text{nm}$, calculate the frequency of small oscillations about $r = r_o$. *Hint: do a Taylor expansion of the potential energy to make it look like a harmonic oscillator for small* $r = r_o$.