PH 125 Quiz 3: Solution

$$W = \int_{r_i}^{r_f} \vec{\mathbf{F}} \cdot \mathrm{d}\vec{\mathbf{r}} = \int_{x_i}^{x_f} F_x \cdot \mathrm{d}x \qquad \Sigma W = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

1. The magnitude of the force on a spring as a function of its displacement from equilibrium ($x = 0$) can be written $F = -kx$, where k is the "force constant" of the spring. What is the amount of work done in stretching the spring from $x = 0$ to $x = x_f$?

Solution: We need only integrate the force through the displacement. If the displacement is purely along the x axis from $x=0$ to x_f, we can write an incremental displacement as $d\vec{\mathbf{x}} = dx\,\hat{\boldsymbol{\imath}}$, whereas the force can be written $\vec{\mathbf{F}} = -kx\,\hat{\boldsymbol{\imath}}$, acting in the opposite direction as the displacement.

$$W = \int_0^{x_f} \vec{\mathbf{F}} \cdot d\vec{\mathbf{x}} = \int_0^{x_f} (-kx\,\hat{\boldsymbol{\imath}}) \cdot (dx\,\hat{\boldsymbol{\imath}}) = \int_0^{x_f} -kx\,dx = \left[-\frac{1}{2}kx^2\right]_0^{x_f} = -\frac{1}{2}kx_f^2$$

2. What is the work done when a 3 kg object free-falls 1 m straight down, relative to the earth's surface? You can neglect air resistance, and let $g = 10\,\mathrm{m/s^2}$.

Solution: The work done by gravity is just the net vertical displacement times the object's weight:

$$W_g = mg\Delta y = (3\,\mathrm{kg})\left(10\,\mathrm{m/s^2}\right)(1\,\mathrm{m}) = 30\,\mathrm{J}$$

3. If you did *not* ignore air resistance in question 2, which of the following would be true?

- ☐ The work done would be more, work is done against air resistance and gravity.
- ☐ The work done would be less, air resistance is countering work by gravity.
- ■ The work done would be the same, the force of air resistance does no work.
- ☐ Cannot be determined without knowing the precise nature of the force of air resistance.

Solution: The work done by *gravity* is exactly the same, since the force of gravity itself does not change and neither does the total distance fallen.

A more formal answer would be that air resistance doesn't do any work, since it is not a force acting through a point of displacement, but a force acting over the whole object itself. We'll get in to that.