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Quiz 1: Solution

1. The position x as a function of time t of a particle traveling along a straight line can be described by the function

$$x(t) = 2.0 + 4.0t - 4.9t^2$$

with $t \geq 0$, x in meters, and t in seconds. At what time is the position maximum? **No sketch is required.**

Double Check possibilities: what about the second derivative?

Given: position as a function of time, $x(t)$.

Find: the time $t > 0$ at which the maximal position occurs, *i.e.*, the maximum of $x(t)$.

Sketch: We can plot the function $x(t)$ to see if indeed there is a maximum for $t > 0$, and roughly estimate its value.

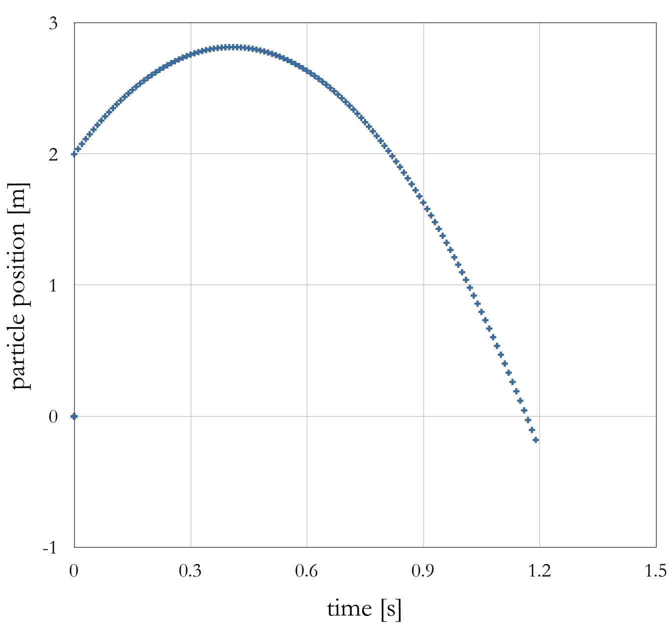

Figure 1: *Position versus time, problem 1.*

Indeed, there is a maximum, somewhere around $t \approx 0.4$ s.

Relevant equations: We need to find the maximum of $x(t)$. Therefore, we need to set the first derivative of x with respect to t equal to zero. We must also check that the second derivative is negative to ensure that we have found a maximum, not a minimum. Therefore, only two equations are needed:

$$\frac{dx}{dt} = \frac{d}{dt}\left[x(t)\right] = 0 \qquad \text{and} \quad \frac{d^2x}{dt^2} = \frac{d^2}{dt^2}\left[x(t)\right] < 0 \quad \Longrightarrow \quad \text{maximum in } x(t)$$

Symbolic solution:

$$\begin{aligned} \frac{dx}{dt} &= \frac{d}{dt}\left[2.0 + 4.0t - 4.9t^2\right] = 4.0 - 9.8t = 0 \\ 9.8t &= 4.0 \\ \Longrightarrow \quad t &= 0.40816\,\text{s} \xrightarrow[\text{digits}]{\text{sign.}} 0.41\,\text{s} \end{aligned}$$

Thus, $x(t)$ takes on an extreme value at $t \approx 0.41$ s. Checking the second derivative:

$$\frac{d^2x}{dt^2} = \frac{d}{dt}\left[4.0 - 9.8t\right] = -9.8 < 0$$

Since $\frac{d^2x}{dt^2}$ is negative at $t \approx 0.41$ s (and indeed for all times), we have found a maximum.

Numeric solution: In this particular case, solving the equation symbolically already leads to the numerical answer.

Double check: From the plot above, we can estimate graphically that the maximum must be somewhere near $t \approx 0.4$ s, consistent with our numerical solution. Graphical analysis is consistent with our analytic solution.

We also know that the maximum for a parabola must be halfway between the two roots. The roots are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4.0 \pm \sqrt{16 + 39.2}}{9.8} = \{-0.35, 1.17\}$$

Halfway between the two roots is just the average of the two roots, or $(-0.35 + 1.17)/2 \approx 0.41$, consistent with our earlier answer.

The dimensions of our answer are given in the problem, so we know that t is in seconds. Since we solved $dx/dt(t)$ for t, the units must be the same as those given, with t still in seconds – our units are correct.

2. According to Abe Simpson,

> The metric system is the tool of the devil! My car gets forty rods to the hogshead and that's the way I likes it.

If one hogshead is approximately 239 L, one rod is approximately 5.03 m, and $1\,\text{km} = 10^3$ m, what is his mileage in km/L? Note that 30 miles/gallon ≈ 12.75 km/L. **No sketch is required.**

Double Check possibilities: verify your unit conversions explicitly. Should the answer be much larger or smaller than the mileage for an average car?

Given: a fuel economy of 40 rods per hogshead, with distance in rods and volume in hogsheads.
Find: The equivalent economy in kilometers per liter.
Relevant equations: We need several unit conversions:

$$\begin{aligned} 1\,\text{rod} &= 5.03\,\text{m} \\ 1\,\text{km} &= 10^3\,\text{m} \\ 1\,\text{hogshead} &= 239\,\text{L} \end{aligned}$$

Symbolic solution: We need only apply the unit conversions in the proper way, and note that only one significant figure is given:

$$(\text{economy in km/L}) = \frac{40\,\text{rods}}{1\,\text{hogshead}}\left[\frac{5.03\,\text{m}}{1\,\text{rod}}\right]\left[\frac{1\,\text{km}}{10^3\,\text{m}}\right]\left[\frac{1\,\text{hogshead}}{239\,\text{L}}\right]$$

Numeric solution: Evaluating the expression above:

$$(\text{economy in km/L}) = (40)\,(5.03)\left(10^{-3}\right)\left(\frac{1}{239}\right)\,\text{km/L} \approx 8.418\times10^{-4}\,\text{km/L} \xrightarrow[\text{digits}]{\text{sign.}} 8\times10^{-4}\,\text{km/L}$$

Double check: The statement by Abe Simpson was meant to be somewhat ridiculous, so it should not be surprising that the answer is ridiculously low.

Given a distance in rods, a relatively small unit of distance, and a volume in hogshead, a very large unit of volume, we should qualitatively expect the answer to be very, very low – certainly much lower than a normal vehicle.