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Quiz 2: Solutions

You have two vectors:

$$\vec{\mathbf{a}} = 1\,\hat{\imath} + 2\,\hat{\jmath} + 3\,\hat{\mathbf{k}} \qquad \text{and} \qquad \vec{\mathbf{b}} = 3\,\hat{\imath} + 6\,\hat{\jmath} + 9\,\hat{\mathbf{k}}$$

1. Find the scalar product of the two vectors, $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}$

First, remember that the way we have written the vectors above is in *component form*:

$$\vec{\mathbf{a}} = a_x\,\hat{\imath} + a_y\,\hat{\jmath} + a_z\,\hat{\mathbf{k}}$$

where it is clear then that $a_x = 1$, $a_y = 2$, *etc.* The scalar product is then readily calculated with the formulas we derived:

$$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = a_x b_x + a_y b_y + a_z b_z = 3 + 12 + 27 = 42$$

2. Find the vector product of the two vectors, $\vec{\mathbf{a}} \times \vec{\mathbf{b}}$

There are two ways to do this one: the hard way, and the easy way.

The hard way makes use of the vector product formula we derived earlier. It is a bit more complex, but easily remembered as the determinant of a 3x3 matrix:

$$\begin{aligned}\vec{\mathbf{a}} \times \vec{\mathbf{b}} &= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{\mathbf{k}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix}\,\hat{\imath} - \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix}\,\hat{\jmath} + \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix}\,\hat{\mathbf{k}} \\ &= \left(a_y b_z - a_z b_y\right)\,\hat{\imath} + \left(a_z b_x - a_x b_z\right)\,\hat{\jmath} + \left(a_x b_y - a_y b_x\right)\,\hat{\mathbf{k}}\end{aligned}$$

Evaluating this with the given components,

$$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = (18 - 18)\,\hat{\imath} + (9 - 9)\,\hat{\jmath} + (6 - 6)\,\hat{\jmath} = 0$$

The easy way? We first notice that the vector $\vec{\mathbf{b}}$ is just thrice the vector $\vec{\mathbf{a}}$, and remember that scalar multiplication is distributive:

$$\vec{\mathbf{b}} = 3\,\hat{\imath} + 6\,\hat{\jmath} + 9\,\hat{\mathbf{k}} = 3\left(1\,\hat{\imath} + 2\,\hat{\jmath} + 3\,\hat{\mathbf{k}}\right) = 3\vec{\mathbf{a}}$$

Scalar multiplication does not change the orientation of the vector, just its magnitude, so $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ must be parallel, which means their cross product is zero.