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Quiz 3: Solution

1. A projectile is launched on level ground with a velocity of $\vec{\mathbf{v}}_i = 3.00\,\hat{\boldsymbol{\imath}} + 4.00\,\hat{\boldsymbol{\jmath}}$. What is the launch angle θ_i, relative to the x axis?

The angle of launch relative to the x axis is just the angle the velocity vector $\vec{\mathbf{v}}_i$ makes with the x axis. For a generic vector $\vec{\mathbf{a}}$ expressed in cartesian coordinates,

$$\vec{\mathbf{a}} = a_x\,\hat{\boldsymbol{\imath}} + a_y\,\hat{\boldsymbol{\jmath}}$$

the angle $\vec{\mathbf{a}}$ makes with the x axis is given by

$$\tan\theta = \frac{a_y}{a_x}$$

In this case, we have $v_y = 4.00$ and $v_x = 3.00$, and thus

$$\theta = \tan^{-1}\left[\frac{v_y}{v_x}\right] = \tan^{-1}\left[\frac{4.00}{3.00}\right] \approx 53.1^\circ$$

2. A particle has a trajectory that follows $\vec{\mathbf{r}} = (3.2\,\hat{\boldsymbol{\imath}} + 1.5\,\hat{\boldsymbol{\jmath}})t + \frac{1}{2}(4.9\,\hat{\boldsymbol{\imath}} + 9.8\,\hat{\boldsymbol{\jmath}})t^2$, where t is in seconds, and r is in meters. What is the velocity in the y direction at $t = 17.2\,\text{s}$?

The velocity vector can be found by differentiating $\vec{\mathbf{r}}$ with respect to t:

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \frac{d}{dt}\left[(3.2\,\hat{\boldsymbol{\imath}} + 1.5\,\hat{\boldsymbol{\jmath}})t + \frac{1}{2}(4.9\,\hat{\boldsymbol{\imath}} + 9.8\,\hat{\boldsymbol{\jmath}})t^2\right] = [3.2\,\hat{\boldsymbol{\imath}} + 1.5\,\hat{\boldsymbol{\jmath}}] + [4.9\,\hat{\boldsymbol{\imath}} + 9.8\,\hat{\boldsymbol{\jmath}}]\,t$$

We want only the y component (the $\hat{\boldsymbol{\jmath}}$ part), so we first collect like terms:

$$\vec{\mathbf{v}} = [3.2 + 4.9t]\;\hat{\boldsymbol{\imath}} + [1.5 + 9.8t]\;\hat{\boldsymbol{\jmath}}$$

This immediately gives us the y component of the velocity v_y just by inspection (or by finding $\vec{\mathbf{v}} \cdot \hat{\boldsymbol{\jmath}}$):

$$v_y = 1.5 + 9.8t$$

Finally, we are asked to find v_y at $t = 17.2\,\text{s}$:

$$v_y(17.2\,\text{s}) = 1.5 + 9.8\,(17.2) \approx 170\,\text{m/s}$$

You should verify for yourself that the units work out correctly in this case ☺