NAME & ID

UNIVERSITY OF ALABAMA
Department of Physics and Astronomy

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Quiz 6: Solutions

1. A block of mass m is connected to a spring with force constant k. If the spring is compressed by an amount x from equilibrium and released, what is the speed of the block as it passes through the equilibrium position of the spring?

This is a purely one-dimensional problem, and we need only one axis. Let the direction of spring's compression be $-x$, and the spring's equilibrium position $x=0$. After the spring is compressed to position $-x$ and released, its restoring force will do work on the mass and increase its kinetic energy. The block's kinetic energy as it passes through $x=0$ can be related to its initial kinetic energy (the moment of release) and the work done by the spring force:

$$K_f = K_i + W_s$$

Since the block starts at rest, $K_i = 0$. The work done by the spring force is simply related to the distance it was compressed, $W_s = \frac{1}{2}kx^2$. We know the work by the spring force is positive for two reasons: first, the spring is in the end giving kinetic energy to the block, and second, in the final state the spring ends up closer to equilibrium than it started.

Now we just need to relate the work done on the spring to the final kinetic energy of the block as it passes through $x=0$:

$$\begin{aligned} K_f &= K_i + W_s \\ \frac{1}{2}mv^2 &= 0 + \frac{1}{2}kx^2 \\ v &= \sqrt{\frac{k}{m}}x \end{aligned}$$

Mathematically, we should have a $\pm$ sign on the velocity, but we know very well that it must be moving in the $+x$ direction.

2. A pendulum consists of a mass m hanging from a cord of length L. Ignore the mass of the cord and air resistance. If the pendulum is released from an angle θ, what is the speed of the mass when the string is vertical?

If the mass is released from an angle θ, it will fall on a circular path and work will be done by the gravitational force in an amount equal to the pendulum bob's weight times the change in height. Only the displacement in the vertical direction matters when we consider work done for or against gravity.

Let the y axis be vertical, and let the pendulum's rest position be $y=0$. Geometrically, you should be able to convince yourself that the vertical displacement resulting from inclining the pendulum at an angle θ is $L - L\cos\theta$. The gravitational work done is then easily calculated and related to the pendulum's speed at $\theta=0$:

$$mg\left(L - L\cos\theta\right) = \frac{1}{2}mv^2$$
$$2gL\left(1-\cos\theta\right) = v^2$$
$$|v| = \sqrt{2gL\left(1-\cos\theta\right)}$$

We did not bother defining a positive angular or horizontal direction, so we do not worry about the sign of the velocity.