Constants:

$$k_e \equiv 1/4\pi\epsilon_o = 8.98755\times10^{9}\,\mathrm{N\cdot m^2\cdot C^{-2}}$$
$$\epsilon_o = 8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2}$$
$$e = 1.60218\times10^{-19}\,\mathrm{C}$$
$$m_{e^-} = 9.10938\times10^{-31}\,\mathrm{kg}$$
$$m_{p^+} = 1.67262\times10^{-27}\,\mathrm{kg}$$
$$N_A = 6.022\times10^{23}\,\text{things/mol}$$

Basic Equations:

$$0 = ax^2+bx^2+c \Longrightarrow x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$\vec{F}_{\text{centr}} = -\frac{mv^2}{r}\,\hat{\mathbf{r}}\ \text{ Centripetal}$$
$$\frac{d^2x}{dt^2} = -\omega^2 x \qquad \omega = 2\pi f \quad \text{simple harmonic motion}$$

Electric Force & Field:

$$\vec{F}_{12} = k_e\frac{q_1q_2}{r_{12}^2}\,\hat{\mathbf{r}}_{12} = q_2\vec{E}_1 \qquad \vec{r}_{12} = \vec{r}_1-\vec{r}_2$$
$$\vec{E}_1 = \vec{F}_{12}/q_2 = k_e\frac{q_1}{r_{12}^2}\,\hat{\mathbf{r}}_{12}$$
$$\vec{E} = k_e\sum_i\frac{q_i}{r_i^2}\,\hat{\mathbf{r}}_i \rightarrow k_e\int\frac{dq}{r^2}\,\hat{\mathbf{r}} = k_e\int_V\frac{\rho\hat{\mathbf{r}}}{r^2}\,d\tau$$
$$\rho\,d\tau \rightarrow \sigma\,da \rightarrow \lambda\,dl$$
$$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_o \qquad \vec{E} = -\vec{\nabla}V$$
$$\int_V\vec{\nabla}\cdot\vec{E}\,d\tau = \oint_S\vec{E}\cdot d\vec{a} = \frac{1}{\epsilon_o}\int_V\rho\,d\tau = \frac{q_{\text{encl}}}{\epsilon_o}$$
$$\left(\vec{E}_{\text{above}}-\vec{E}_{\text{below}}\right)\cdot\hat{\mathbf{n}} = 4\pi k_e\sigma = \sigma/\epsilon_o \quad \text{sheet of charge with } \sigma$$
$$F_{\text{sheet}} = \frac{\sigma}{2}\left(\vec{E}_{\text{above}}+\vec{E}_{\text{below}}\right)$$

Electric Potential (static case!):

$$\Delta V = V_B - V_A = \frac{\Delta U}{q} = -\int_A^B\vec{E}\cdot d\vec{l}$$
$$V_{\text{point}} = k_e\frac{q}{r} \rightarrow V_{\text{continuous}} = k_e\int\frac{dq}{r} = k_e\int\frac{\rho}{r}\,d\tau$$
$$U_{\text{pair of point charges}} = k_e\frac{q_1q_2}{r_{12}} = V_1q_2 = V_2q_1$$
$$U_{\text{system}} = \text{sum over unique pairs} = \sum_{\text{pairs } ij}\frac{k_eq_iq_j}{r_{ij}} \rightarrow \frac{1}{2}\int\rho V\,d\tau$$
$$U_{\text{field}} = \frac{\epsilon_o}{2}\int E^2\,d\tau = \frac{1}{2}\int\rho V\,d\tau$$
$$\frac{\partial V}{\partial n} = \vec{\nabla}V\cdot\hat{\mathbf{n}} = -\vec{E}\cdot\hat{\mathbf{n}} \qquad \text{boundary condition, sheet of chg}$$
$$\frac{\partial V}{\partial n} = -\sigma/\epsilon_o \qquad \text{boundary condition, cond.}$$

Vectors:

$$|\vec{F}| = \sqrt{F_x^2+F_y^2}\ \text{ magnitude} \qquad \theta = \tan^{-1}\left[\frac{F_y}{F_x}\right]\ \text{ direction}$$
$$\hat{\mathbf{r}} = \vec{r}/|\vec{r}| \quad \text{construct any unit vector}$$
$$\text{let}\quad \vec{a} = a_x\hat{\mathbf{x}}+a_y\hat{\mathbf{y}}+a_z\hat{\mathbf{z}} \quad\text{and}\quad \vec{b} = b_x\hat{\mathbf{x}}+b_y\hat{\mathbf{y}}+b_z\hat{\mathbf{z}}$$
$$\vec{a}\cdot\vec{b} = a_xb_x+a_yb_y+a_zb_z = \sum_{i=1}^{n}a_ib_i = |\vec{a}||\vec{b}|\cos\theta$$
$$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$$
$$\vec{a}\times\vec{b} = \begin{vmatrix}\hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix} = (a_yb_z-a_zb_y)\,\hat{\mathbf{x}}+(a_zb_x-a_xb_z)\,\hat{\mathbf{y}}+(a_xb_y-a_yb_x)\,\hat{\mathbf{z}}$$

Vector Calculus:

$$d\vec{l} = \hat{\mathbf{x}}\,dx+\hat{\mathbf{y}}\,dy+\hat{\mathbf{z}}\,dz \quad \text{length, cartesian}$$
$$d\vec{l} = \hat{\mathbf{r}}\,dr+\hat{\boldsymbol{\theta}}\,r\,d\theta+\hat{\boldsymbol{\varphi}}\,r\sin\theta\,d\varphi \quad \text{length, spherical}$$
$$d\tau = dx\,dy\,dz \quad \text{volume, cartesian}$$
$$d\tau = r^2\sin\theta\,dr\,d\theta\,d\varphi \quad \text{volume, spherical}$$
$$\vec{\nabla} = \hat{\mathbf{x}}\frac{\partial}{\partial x}+\hat{\mathbf{y}}\frac{\partial}{\partial y}+\hat{\mathbf{z}}\frac{\partial}{\partial z} \quad \text{cartesian}$$
$$\vec{\nabla} = \hat{\mathbf{r}}\frac{\partial}{\partial r}+\hat{\boldsymbol{\theta}}\frac{1}{r}\frac{\partial}{\partial\theta}+\hat{\boldsymbol{\varphi}}\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi} \quad \text{spherical}$$
$$\vec{\nabla}\cdot\vec{F} = \frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z} \quad \text{cartesian}$$
$$\vec{\nabla}\cdot\vec{F} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2F_r\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,F_\theta\right)+\frac{1}{r\sin\theta}\frac{\partial F_\varphi}{\partial\varphi} \quad \text{spherical}$$

Table 1: *Relationships between cartesian and polar coordinates.*

cartesian (x, y)	**polar** (r, θ)
$r = \sqrt{x^2+y^2}$	$x = r\cos\theta$
$\theta = \tan^{-1}(y/x)$	$y = r\sin\theta$

Table 2: *Relationships between cartesian and spherical coordinates.*

	cartesian (x, y, z)	**spherical** $(r, \varphi\vert_0^{2\pi}, \theta\vert_0^{\pi})$
(x, y, z)	$x = x$	$x = r\sin\theta\cos\varphi$
	$y = y$	$y = r\sin\theta\sin\varphi$
	$z = z$	$z = r\cos\theta$
(r, θ, φ)	$r = \sqrt{x^2+y^2+z^2}$	$r = r$
	$\varphi = \tan^{-1}\left(\frac{y}{x}\right)$	$\varphi = \varphi$
	$\theta = \tan^{-1}\left(\sqrt{x^2+y^2}/z\right)$	$\theta = \theta$

Table 3: *Unit vectors in cartesian and spherical systems.*

	spherical
cartesian	$\hat{\mathbf{x}} = \sin\theta\cos\varphi\,\hat{\mathbf{r}}+\cos\theta\cos\varphi\,\hat{\boldsymbol{\theta}}-\sin\varphi\,\hat{\boldsymbol{\varphi}}$
	$\hat{\mathbf{y}} = \sin\theta\sin\varphi\,\hat{\mathbf{r}}+\cos\theta\sin\varphi\,\hat{\boldsymbol{\theta}}+\cos\varphi\,\hat{\boldsymbol{\varphi}}$
	$\hat{\mathbf{z}} = \cos\theta\,\hat{\mathbf{r}}-\sin\theta\,\hat{\boldsymbol{\theta}}$
	cartesian
spherical	$\hat{\mathbf{r}} = \frac{1}{r}(x\,\hat{\mathbf{x}}+y\,\hat{\mathbf{y}}+z\,\hat{\mathbf{z}})$
	$\hat{\mathbf{r}} = \sin\theta\cos\varphi\,\hat{\mathbf{x}}+\sin\theta\sin\varphi\,\hat{\mathbf{y}}+\cos\theta\,\hat{\mathbf{z}}$
	$\hat{\boldsymbol{\theta}} = \cos\theta\cos\varphi\,\hat{\mathbf{x}}+\cos\theta\sin\varphi\,\hat{\mathbf{y}}-\sin\theta\,\hat{\mathbf{z}}$
	$\hat{\boldsymbol{\varphi}} = -\sin\varphi\,\hat{\mathbf{x}}+\cos\varphi\,\hat{\mathbf{y}}$

Calculus of possible utility:

$$\int \frac{1}{x}\,dx = \ln x + c \tag{1}$$

$$\int u\,dv = uv - \int v\,du \tag{2}$$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + c \tag{3}$$

$$\int \frac{x}{a^2+x^2}\,dx = \frac{1}{2}\ln|a^2+x^2| + c \tag{4}$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a} + C \tag{5}$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}}\,dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C \tag{6}$$

$$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}} + C \tag{7}$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax + C \tag{8}$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax + C \tag{9}$$

$$\frac{d}{dx}\tan x = \sec^2 x = \frac{1}{\cos^2 x} \tag{10}$$

$$\tag{11}$$