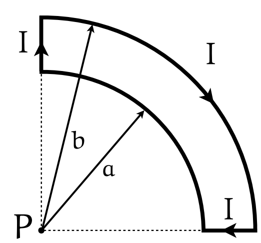
Name & CWID ____

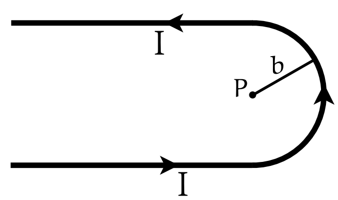
PH126 Exam II

Instructions

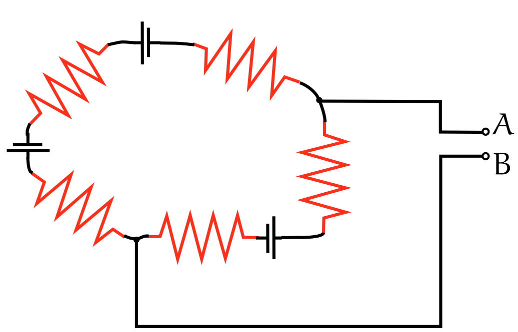
1. **Solve three of the six problems below.** All problems have equal weight.
2. Clearly mark your which problems you have chosen using the tick box.
3. You are allowed 1 sheet of standard 8.5x11 in paper and a calculator.



□ **1.** Find the magnetic field at point P for the steady current configuration above.



□ **2.** Find the magnetic field at point P for the steady current configuration above, in which a long wire is bent into a hairpin shape. The point P lies at the center of the half-circle.



□ **3.** In the circuit above, all five resistors have the same value, 100 Ω, and each battery has a rated voltage of 1.5 V and no internal resistance. Find the open-circuit voltage and the short-circuit current for the terminals A, B. Then find the Thèvenin equivalent circuit (i.e., the ideal battery and resistor that could replace this circuit between terminals A, B.)

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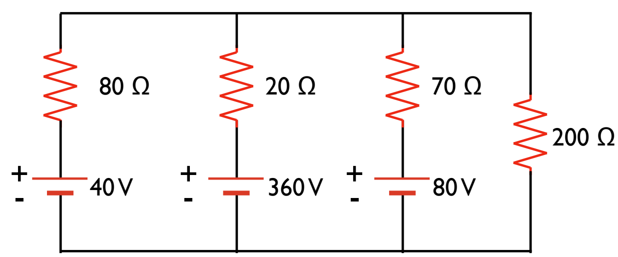
□ **4.** A wire carrying current I runs down the y axis and to the origin, thence out to infinity along the positive x axis. Show that the magnetic field in the quadrant $x > 0$, $y > 0$ of the xy plane is given by

$$B_z = \frac{\mu_o I}{4\pi}\left(\frac{1}{x} + \frac{1}{y} + \frac{x}{y\sqrt{x^2+y^2}} + \frac{y}{x\sqrt{x^2+y^2}}\right)$$

5. In the circuit below, determine the current in each resistor and the voltage across the $200\,\Omega$ resistor.



6. You are given two batteries, one of $9\,\text{V}$ and internal resistance $0.50\,\Omega$, and another of $3\,\text{V}$ and internal resistance $0.40\,\Omega$. How must these batteries be connected to give the largest possible current through an external $0.30\,\Omega$ resistor? What is this current?

Constants:

$$k_e \equiv 1/4\pi\epsilon_o = 8.98755 \times 10^9\ \mathrm{N \cdot m^2 \cdot C^{-2}}$$
$$\epsilon_o = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$$
$$\mu_o \equiv 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$$
$$c^2 = 1/\mu_o\epsilon_o$$
$$e = 1.60218 \times 10^{-19}\ \mathrm{C}$$

Basic Equations / Mechanics:

$$0 = ax^2 + bx^2 + c \Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\vec{F}_{\text{centr}} = -\frac{mv^2}{r}\,\hat{\mathbf{r}} \quad \text{Centripetal}$$

Electric Force & Field (static case):

$$\vec{F}_{12} = k_e \frac{q_1 q_2}{r_{12}^2}\,\hat{\mathbf{r}}_{12} = q_2 \vec{E}_1 \qquad \vec{r}_{12} = \vec{r}_1 - \vec{r}_2$$
$$\vec{E}_1 = \vec{F}_{12}/q_2 = k_e \frac{q_1}{r_{12}^2}\,\hat{\mathbf{r}}_{12}$$
$$\vec{E} = k_e \sum_i \frac{q_i}{r_i^2}\,\hat{\mathbf{r}}_i \to k_e \int \frac{dq}{r^2}\,\hat{\mathbf{r}} = k_e \int_V \frac{\rho \hat{\mathbf{r}}}{r^2}\,d\tau$$
$$\rho\,d\tau \to \sigma\,da \to \lambda\,dl$$
$$\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_o \qquad \vec{E} = -\vec{\nabla} V$$

Electric Potential (static case):

$$\Delta V = V_B - V_A = \frac{\Delta U}{q} = -\int_A^B \vec{E} \cdot d\vec{l}$$
$$V_{\text{point}} = k_e \frac{q}{r} \to V_{\text{continuous}} = k_e \int \frac{dq}{r} = k_e \int \frac{\rho}{r}\,d\tau$$
$$U_{\text{pair of point charges}} = k_e \frac{q_1 q_2}{r_{12}} = V_1 q_2 = V_2 q_1$$
$$U_{\text{system}} = \text{sum over unique pairs} = \sum_{\text{pairs } ij} \frac{k_e q_i q_j}{r_{ij}} \to \frac{1}{2}\int \rho V\,d\tau$$
$$U_{\text{field}} = \frac{\epsilon_o}{2}\int E^2\,d\tau = \frac{1}{2}\int \rho V\,d\tau$$

Current & Resistance:

$$I = \int_S \vec{J} \cdot d\vec{A} \xrightarrow{\text{uniform J}} I = \frac{dQ}{dt} = nqAv_d$$
$$J = \sum_k n_k q_k v_k \xrightarrow{\text{uniform J}} J = \frac{I}{A} = nqv_d$$
$$\int_S \vec{J} \cdot d\vec{A} = -\frac{d}{dt}\int_V \rho\,dV_{ol}$$
$$R = \frac{\rho l}{A} \qquad \rho = 1/\sigma$$
$$\vec{v}_d = \frac{q\tau}{m}\vec{E} \qquad \tau = \text{scattering time}$$
$$\rho = 1/\sigma = \frac{m}{nq^2\tau}$$
$$R = \Delta V/I \quad \text{Ohm}$$
$$E = \rho J \quad \text{or} J = \sigma E \quad \text{Ohm}$$
$$\mathscr{P} = dU/dt = I\Delta V \quad \text{power}$$
$$R_{\text{eq}} = R_1 + R_2 + \ldots \quad \text{series}$$
$$1/R_{\text{eq}} = 1/R_1 + 1/R_2 + \ldots \quad \text{parallel}$$
$$\sum I_{\text{in}} = \sum I_{\text{out}} \quad \text{junction}$$
$$\sum_{\text{closed path}} \Delta V = 0 \quad \text{loop}$$

Magnetism:

$$\vec{F}_B = q\vec{v} \times \vec{B}$$
$$d\vec{F}_B = I\,d\vec{l} \times \vec{B} \quad \text{I-carrying wire}$$
$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{I\,d\vec{l} \times \hat{\mathbf{r}}}{r^2} \quad \text{wire}$$
$$\frac{F_B}{l} = \frac{\mu_o I_1 I_2}{2\pi d} \quad \text{2 wires}$$
$$\vec{\tau} = \vec{\mu} \times \vec{B} \quad \text{with} \quad \vec{\mu} = I\vec{A} \quad \text{I loop}$$
$$U = -\vec{\mu} \times \vec{B}$$
$$B = \frac{\mu_o I}{4\pi d}(\sin\theta_2 - \sin\theta_1) \quad \text{finite straight wire}$$

Vector Calculus:

$$d\vec{l} = \hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz = \hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\theta}}\,r\,d\theta + \hat{\boldsymbol{\varphi}}\,r\sin\theta\,d\varphi \quad \text{cartesian, spherical}$$
$$d\tau = dx\,dy\,dz \quad = r^2\sin\theta\,dr\,d\theta\,d\varphi \quad \text{cartesian, spherical}$$
$$\vec{\nabla} = \hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z} \quad \text{cartesian}$$
$$\vec{\nabla} = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\boldsymbol{\theta}}\frac{1}{r}\frac{\partial}{\partial\theta} + \hat{\boldsymbol{\varphi}}\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi} \quad \text{spherical}$$
$$\vec{\nabla} \cdot \vec{F} = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \quad \text{cartesian}$$
$$\vec{\nabla} \cdot \vec{F} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 F_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\,F_\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\varphi}{\partial\varphi} \quad \text{spherical}$$

	cartesian (x, y, z)
spherical	$r = \sqrt{x^2 + y^2 + z^2}$ $\varphi = \tan^{-1}\left(\frac{y}{x}\right)$ $\theta = \tan^{-1}\left(\sqrt{x^2 + y^2}/z\right)$
	spherical $(r, \varphi\vert_0^{2\pi}, \theta\vert_0^{\pi})$
cartesian	$x = r\sin\theta\cos\varphi$ $y = r\sin\theta\sin\varphi$ $z = r\cos\theta$

	cartesian $(x, y, z,)$
spherical	$\hat{\mathbf{r}} = \frac{1}{r}(x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}})$ $\hat{\mathbf{r}} = \sin\theta\cos\varphi\,\hat{\mathbf{x}} + \sin\theta\sin\varphi\,\hat{\mathbf{y}} + \cos\theta\,\hat{\mathbf{z}}$ $\hat{\boldsymbol{\theta}} = \cos\theta\cos\varphi\,\hat{\mathbf{x}} + \cos\theta\sin\varphi\,\hat{\mathbf{y}} - \sin\theta\,\hat{\mathbf{z}}$ $\hat{\boldsymbol{\varphi}} = -\sin\varphi\,\hat{\mathbf{x}} + \cos\varphi\,\hat{\mathbf{y}}$
	spherical $(r, \varphi\vert_0^{2\pi}, \theta\vert_0^{\pi})$
cartesian	$\hat{\mathbf{x}} = \sin\theta\cos\varphi\,\hat{\mathbf{r}} + \cos\theta\cos\varphi\,\hat{\boldsymbol{\theta}} - \sin\varphi\,\hat{\boldsymbol{\varphi}}$ $\hat{\mathbf{y}} = \sin\theta\sin\varphi\,\hat{\mathbf{r}} + \cos\theta\sin\varphi\,\hat{\boldsymbol{\theta}} + \cos\varphi\,\hat{\boldsymbol{\varphi}}$ $\hat{\mathbf{z}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}}$

Calculus of possible utility:

$$\int \frac{1}{x}\,dx = \ln x + c$$
$$\int u\,dv = uv - \int v\,du$$
$$\int \frac{1}{1 + x^2}\,dx = \tan^{-1} x + c$$
$$\int \frac{x}{a^2 + x^2}\,dx = \frac{1}{2}\ln|a^2 + x^2| + c$$
$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a} + C$$
$$\int \frac{1}{\sqrt{x^2 \pm a^2}}\,dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$$
$$\int \frac{dx}{(a^2 + x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2 + x^2}} + C$$
$$\frac{d}{dx}\tan x = \sec^2 x = \frac{1}{\cos^2 x}$$
$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$
$$\frac{d}{dx}\frac{1}{u} = \frac{-1}{u^2}\frac{du}{dx}$$