

Constants:

$$\begin{aligned} k_e &\equiv 1/4\pi\epsilon_0 = 8.98755 \times 10^9 \text{ N} \cdot \text{m}^2 \cdot \text{C}^{-2} \\ \epsilon_0 &= 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2 \\ \mu_0 &\equiv 4\pi \times 10^{-7} \text{ T} \cdot \text{m}/\text{A} \\ c^2 &= 1/\mu_0\epsilon_0 \approx (3 \times 10^8 \text{ m/s})^2 \\ e &= 1.60218 \times 10^{-19} \text{ C} \end{aligned}$$

Maxwell's equations, Lorentz, & continuity (without polarization or magnetization)

$$\begin{aligned} \vec{F}_B &= q\vec{v} \times \vec{B} \\ \vec{\nabla} \cdot \vec{E} &= \frac{\rho}{\epsilon_0} \\ \vec{\nabla} \cdot \vec{B} &= 0 \\ \vec{\nabla} \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \vec{\nabla} \times \vec{B} &= \mu_0 \vec{j} + \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} \\ \partial \rho / \partial t + \vec{\nabla} \cdot \vec{j} &= 0 \\ \oint_S \vec{E} \cdot d\vec{A} &= \frac{q}{\epsilon_0 \epsilon_r} = \frac{1}{\epsilon_0} \int_V \rho \, dV \\ \oint_S \vec{B} \cdot d\vec{A} &= 0 \\ \oint_C \vec{E} \cdot d\vec{l} &= -\frac{\partial}{\partial t} \int_S \vec{B} \cdot d\vec{A} \\ \oint_C \vec{B} \cdot d\vec{l} &= \mu_0 \int_S \vec{j} \cdot d\vec{A} + \epsilon_0 \mu_0 \frac{\partial}{\partial t} \int_S \vec{E} \cdot d\vec{A} \\ \int_S \vec{j} \cdot d\vec{A} &= -\frac{d}{dt} \int_V \rho \, dV \\ \text{Note } c^2 &= \frac{1}{\mu_0 \epsilon_0} \end{aligned}$$

Electric Force & Field (static case):

$$\begin{aligned} \vec{F}_{12} &= k_e \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12} = q_2 \vec{E}_1 \quad \vec{r}_{12} = \vec{r}_1 - \vec{r}_2 \\ \vec{E}_1 &= \vec{F}_{12}/q_2 = k_e \frac{q_1}{r_{12}^2} \hat{r}_{12} \\ \vec{E} &= k_e \sum_i \frac{q_i}{r_i^2} \hat{r}_i \rightarrow k_e \int \frac{dq}{r^2} \hat{r} = k_e \int_V \frac{\rho \hat{r}}{r^2} d\tau \\ \rho \, d\tau &\rightarrow \sigma \, da \rightarrow \lambda \, dl \\ \vec{\nabla} \cdot \vec{E} &= \rho/\epsilon_0 \quad \vec{E} = -\vec{\nabla} V \end{aligned}$$

Electric Potential (static case):

$$\begin{aligned} \Delta V &= V_B - V_A = \frac{\Delta U}{q} = -\int_A^B \vec{E} \cdot d\vec{l} \\ V_{\text{point}} &= k_e \frac{q}{r} \rightarrow V_{\text{continuous}} = k_e \int \frac{dq}{r} = k_e \int \frac{\rho}{r} d\tau \\ U_{\text{pair of point charges}} &= k_e \frac{q_1 q_2}{r_{12}} = V_1 q_2 = V_2 q_1 \\ U_{\text{field}} &= \frac{\epsilon_0}{2} \int E^2 d\tau = \frac{1}{2} \int \rho V d\tau \end{aligned}$$

Field transformations

(primed frame moves with v relative to unprimed along x)

$$\begin{aligned} E'_x &= E_x & E'_y &= \gamma(E_y - vB_z) & E'_z &= \gamma(E_z + vB_y) \\ B'_x &= B_x & B'_y &= \gamma(B_y + \frac{v}{c^2} E_z) & B'_z &= \gamma(B_z - \frac{v}{c^2} E_y) \end{aligned}$$

Current & Resistance:

$$\begin{aligned} I &= \int_S \vec{j} \cdot d\vec{A} \xrightarrow{\text{uniform } j} I = \frac{dQ}{dt} = nqAv_d \\ j &= \sum_k n_k q_k v_k \xrightarrow{\text{uniform } j} j = \frac{I}{A} = nqv_d \\ R &= \Delta V/I \quad \text{or } \vec{j} = \sigma \vec{E} \quad \text{Ohm, isotropic} \\ j_i &= \sum_{j=1}^3 \sigma_{ij} E_j \quad \text{Ohm, anisotropic/general} \end{aligned}$$

Vector Calculus:

$$\begin{aligned} d\vec{l} &= \hat{x} dx + \hat{y} dy + \hat{z} dz = \hat{r} dr + \hat{\theta} r d\theta + \hat{\phi} r \sin \theta d\phi \quad \text{cartesian, spherical} \\ d\tau &= dx dy dz = r^2 \sin \theta dr d\theta d\phi \quad \text{cartesian, spherical} \\ \vec{\nabla} &= \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z} \quad \text{cartesian} \\ \vec{\nabla} &= \hat{r} \frac{\partial}{\partial r} + \hat{\theta} \frac{1}{r} \frac{\partial}{\partial \theta} + \hat{\phi} \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} \quad \text{spherical} \\ \vec{\nabla} \cdot \vec{F} &= \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \quad \text{cartesian} \\ \vec{\nabla} \cdot \vec{F} &= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 F_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta F_\theta) + \frac{1}{r \sin \theta} \frac{\partial F_\phi}{\partial \phi} \quad \text{spherical} \\ \vec{\nabla} \times \vec{v} &= \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta v_\phi) - \frac{\partial v_\theta}{\partial \phi} \right] \hat{r} + \frac{1}{r} \left[\frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r} (rv_\phi) \right] \hat{\theta} \\ &\quad + \frac{1}{r} \left[\frac{\partial}{\partial r} (rv_\theta) - \frac{\partial v_r}{\partial \theta} \right] \hat{\phi} \quad \text{spherical} \end{aligned}$$

| | cartesian (x, y, z) |
|-----------|--|
| spherical | $r = \sqrt{x^2 + y^2 + z^2}$ |
| | $\phi = \tan^{-1} \left(\frac{y}{x} \right)$ |
| | $\theta = \tan^{-1} \left(\sqrt{x^2 + y^2} / z \right)$ |
| | spherical (r, $\phi _0^{2\pi}$, $\theta _0^\pi$) |
| cartesian | $x = r \sin \theta \cos \phi$ |
| | $y = r \sin \theta \sin \phi$ |
| | $z = r \cos \theta$ |

| | cartesian (x, y, z,) |
|-----------|---|
| spherical | $\hat{r} = \frac{1}{r} (x \hat{x} + y \hat{y} + z \hat{z})$ |
| | $\hat{\theta} = \cos \theta \cos \phi \hat{x} + \cos \theta \sin \phi \hat{y} - \sin \theta \hat{z}$ |
| | $\hat{\phi} = -\sin \phi \hat{x} + \cos \phi \hat{y}$ |
| | |
| | spherical (r, $\phi _0^{2\pi}$, $\theta _0^\pi$) |
| cartesian | $\hat{x} = \sin \theta \cos \phi \hat{r} + \cos \theta \cos \phi \hat{\theta} - \sin \phi \hat{\phi}$ |
| | $\hat{y} = \sin \theta \sin \phi \hat{r} + \cos \theta \sin \phi \hat{\theta} + \cos \phi \hat{\phi}$ |
| | $\hat{z} = \cos \theta \hat{r} - \sin \theta \hat{\theta}$ |

Relativity

$$\begin{aligned} \gamma &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \\ \Delta t'_{\text{moving}} &= \gamma \Delta t_{\text{stationary}} = \gamma \Delta t_p \\ L'_{\text{moving}} &= \frac{L_{\text{stationary}}}{\gamma} = \frac{L_p}{\gamma} \\ x' &= \gamma(x - vt) \\ \Delta t' &= t'_1 - t'_2 = \gamma \left(\Delta t - \frac{v \Delta x}{c^2} \right) \\ p &= \gamma m v \\ v_{\text{obj}} &= \frac{v + v'_{\text{obj}}}{1 + \frac{v v'_{\text{obj}}}{c^2}} & v'_{\text{obj}} &= \frac{v_{\text{obj}} - v}{1 - \frac{v v_{\text{obj}}}{c^2}} \end{aligned}$$