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PH126 Final Exam

Take-home exam agreement: The work submitted for this exam is entirely my own. I have used only my textbook, notes, and a calculator.

Signed:____________________________________

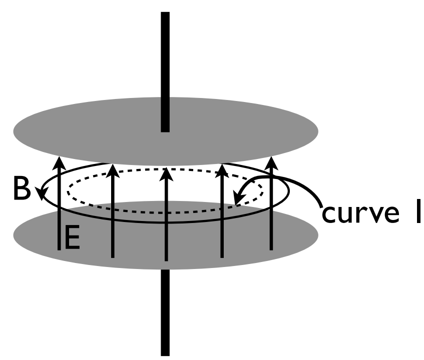
Instructions

1. Solve all problems below. All problems have equal weight.
2. You may use your textbook and notes.
3. You may not collaborate.
4. Show as much work as possible for partial credit. **No work = no credit.**
5. Solve the problems on separate sheets. Staple your sheets to the exam.
6. Return the exam to Dr. LeClair by **5pm on Thurs 10 Dec 2009**.

1. Suppose we have a parallel plate capacitor connected to an ac generator of relatively low frequency ω. You know that electric field deep inside the capacitor (i.e., ignoring edge effects) can be written as

$$E = E_o e^{i\omega t} \tag{1}$$

where E_o is a constant.



Will this work indefinitely as frequency goes up? Not really; as the electric field oscillates, that creates a time-varying flux through loops like curve 1 shown below, which will produce a magnetic field. We can find that field with Maxwell's equations, using curve 1 as our boundary of integration:

$$c^2 \oint_1 \vec{B} \cdot d\vec{s} = \frac{\partial}{\partial t} \int_1 \vec{E} \cdot \vec{n}\, da \tag{2}$$

$$2\pi r c^2 B = \frac{\partial}{\partial t} \pi r^2 E \tag{3}$$

$$\implies \quad B = \frac{i\omega r}{2c^2} E_o e^{i\omega t} \tag{4}$$

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Thus, as you know, there is an oscillating magnetic field in the capacitor as well, meaning at high enough frequencies it has a little bit of inductance. Now we have a problem, however: if there is an oscillating magnetic field inside, as given above, then this must induce an additional *electric* field in addition to the applied electric field. Thus, the total field will be the original one plus that induced by the oscillating magnetic field, which was itself due to the original field! This continues on indefinitely in fact, since the new additional electric field will create a new oscillating magnetic field, and so on.

(a) Using the integral form of Faraday's law, find the first "correction" to the electric field, viz., that due to the oscillating magnetic field given above.

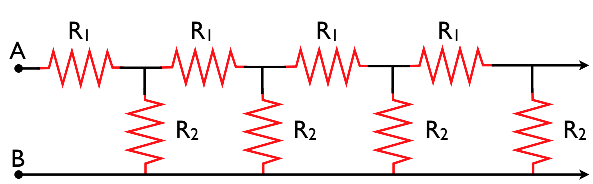
(b) Now that you have the "corrected" electric field, the expression for the magnetic field is not quite right. Find the second term in the magnetic field expression.

(c) Sketch the electric field along the radial direction (i.e., perpendicular to the vertical axis) after the first correction, still ignoring edge effects. You should find that the field is no longer uniform after correcting for the time-variation of the fields.

Incidentally, if you continue correcting the fields indefinitely, until you have a fully self-consistent field (written as an infinite series), you will get the original electric field times what is known as a Bessel function. The first correction term plus the original field is a very good approximation to the real field, however.

2. A flat circular disk with radius R carries a uniform surface charge density σ. It rotates with an angular velocity ω about the z-axis. Find the magnetic field $B(z)$ at any point z along the rotation axis.

3. Find the input resistance (between terminals A and B) of the following infinite series of resistors.



Show that, if voltage V_o is applied at the input to such a chain, the voltage at successive nodes decreases in a geometric series. What ratio is required for the resistors to make the ladder an attenuator that halves the voltage at every step? Can you suggest a way to terminate the ladder after a few sections without introducing any error in its attenuation? *Hint: If we put another "link" on the left of this infinite chain, we get exactly the same configuration.*

4. Consider the wave in free space

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$$E_x = 0 \qquad E_y = E_o \sin(kx + \omega t) \qquad E_z = 0$$

$$B_x = 0 \qquad B_y = 0 \qquad B_z = -\frac{E_o}{c} \sin(kx + \omega t)$$

Show that this field can satisfy Maxwell's equations if ω and k are related in a certain way. Suppose $\omega = 10^{10}$ Hz and $E_o = 15$ V/m. What is the wavelength in m? What is the energy density, averaged over a large region?

5. We have two point charges connected by a rigid rod, forming a dipole. It is placed in an external electric field $\vec{E}(\vec{r})$.

(a) Suppose that the electric field is uniform: $\vec{E} = \vec{E}_o$ where $\vec{E}_o$ is a constant vector. What will be the total force on the dipole?

(b) Now suppose the field is not uniform, but that it only changes by a small amount over the distance d. Show that the z-component of the total force on the dipole is approximately

$$F_z = p_z \frac{\partial E_z}{\partial z} \tag{5}$$

where p_z is the z-component of the dipole moment $\vec{p}$.

(c) For the case when the dipole has an arbitrary orientation (not necessarily aligned with the z-axis), show that the total vector force on the dipole can be written

$$\vec{F} = \left(\vec{p} \cdot \vec{\nabla}\right) \vec{E} \tag{6}$$

(d) Why is a charged rubber rod able to attract bits of paper without touching them?

6. (a) We wish to find the total relativistic energy of a particle moving with velocity v. We can do this by finding the work required to accelerate a particle from rest to velocity v over some path. Start like this:

$$E = \int \vec{F} \cdot d\vec{l} = \int \frac{d\vec{p}}{dt} \cdot d\vec{l} = \int \frac{d\vec{p}}{dt} \cdot \frac{d\vec{l}}{dt} dt = \int \frac{d\vec{p}}{dt} \cdot \vec{v}\, dt \tag{7}$$

Net, you can use the chain rule in evaluating the time derivative of $\vec{p} \cdot \vec{v}$ to make a clever substitution in the equation above. Continue the derivation and show that $E = \gamma mc^2 - mc^2$.

(b) While you're at it, show that this expression reduces to a rest energy plus the classical kinetic energy for $v \ll c$.

7. Find the attractive force between the conductors of a parallel plate capacitor, if the total charge on each conductor is fixed. Repeat for a coaxial cable.

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8. *An XKCD problem:* "A clever guy has an idea for a date with his girlfriend. He wants to rent a cherry picker, drive it to her door, and pick her up in it. Then, he'd drive to the beach, and get there at just the right time to watch the sun set. Once the sun had set, he'd activate the cherry picker, they'd be lifted up above the beach . . . and they'd watch the sun set again. Clearly, this is an excellent idea, and any girl would be lucky to see this guy at her door. But is it plausible? How fast and how high does the cherry picker have to go?" – Randall Munroe