University of Alabama
Department of Physics and Astronomy

PH 126 / LeClair Fall 2009

Problem Set 1: Math review

Instructions:

1. Answer all questions below.
2. Some problems have different due dates!
3. You may collaborate, but everyone must turn in their own work

The following four problems are due Wed 26 August 2009 by the end of the day.

1. Find the separation vector $\vec{\boldsymbol{\mathscr{r}}} = \vec{\mathbf{r}} - \vec{\mathbf{r}}'$ from the source point $\vec{\mathbf{r}}' = (3,4,5)$ to the field point $\vec{\mathbf{r}} = (7,2,17)$. Determine its magnitude $|\vec{\boldsymbol{\mathscr{r}}}|$ and construct the corresponding unit vector $\hat{\boldsymbol{\mathscr{r}}}$.

2. Find the angle between the *body* diagonals of a cube. Use one of the vector products.

3. If $\vec{\mathbf{a}} = \hat{\mathbf{x}} - \hat{\mathbf{y}} + \hat{\mathbf{z}}$, $\vec{\mathbf{b}} = 2\hat{\mathbf{x}} - \hat{\mathbf{y}}$, and $\vec{\mathbf{c}} = 3\hat{\mathbf{x}} + 5\hat{\mathbf{y}} - 7\hat{\mathbf{z}}$, verify the identity

$$\vec{\mathbf{a}} \times \left(\vec{\mathbf{b}} \times \vec{\mathbf{c}}\right) = \left(\vec{\mathbf{a}} \cdot \vec{\mathbf{c}}\right) \vec{\mathbf{b}} - \left(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}\right) \vec{\mathbf{c}}$$

4. If $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are given constant vectors and ω is a constant, describe the trajectory of a particle given by $\vec{\mathbf{r}}(t) = \vec{\mathbf{a}}\cos\omega t + \vec{\mathbf{b}}\sin\omega t$. Verify the following

$$\frac{d^2\vec{\mathbf{r}}}{dt^2} + \omega^2\vec{\mathbf{r}} = 0 \qquad \vec{\mathbf{r}} \times \frac{d\vec{\mathbf{r}}}{dt} = \omega\vec{\mathbf{a}} \times \vec{\mathbf{b}} \qquad \left|\frac{d\vec{\mathbf{r}}}{dt}\right|^2 + \omega^2|\vec{\mathbf{r}}|^2 = \omega^2\left(|\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2\right)$$

The following five problems are due Fri 28 August 2009 by the end of the day.

5. If $\vec{\mathbf{F}} = y\hat{\mathbf{x}} - z\hat{\mathbf{y}} + x\hat{\mathbf{z}}$, is there a potential φ such that $\vec{\mathbf{F}} = \vec{\boldsymbol{\nabla}}\varphi$? If so, find a possible φ. How about for $\vec{\mathbf{F}} = y^2\hat{\mathbf{x}} - z^3\hat{\mathbf{y}} + x^4\hat{\mathbf{z}}$, or $\vec{\mathbf{F}} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z^2\hat{\mathbf{z}}$?

6. Find the potential φ so that

$$\vec{\boldsymbol{\nabla}}\varphi = \vec{\mathbf{F}} = \left(y^2 + 2xz^2 - 1\right)\hat{\mathbf{x}} + 2xy\hat{\mathbf{y}} + \left(2x^2z + z^3\right)\hat{\mathbf{z}}$$

7. Sketch the vector function $\vec{\mathbf{v}} = \hat{\mathbf{r}}/r^2$ and compute its divergence. The answer should surprise you. Can you explain it?

8. Calculate the divergence and curl for the following vector functions $\vec{\mathbf{F}}(x, y, z)$. Then verify that the divergence of the curl is zero for each, i.e., $\vec{\boldsymbol{\nabla}} \cdot \left(\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{F}}\right) = 0$.

$$x\hat{\mathbf{x}} + (y+z)\,\hat{\mathbf{y}} + (x+y+z)\,\hat{\mathbf{z}}$$
$$f(x)\hat{\mathbf{x}} + g(y)\hat{\mathbf{y}} + h(z)\hat{\mathbf{z}}$$
$$f(y,z)\hat{\mathbf{x}} + g(z,x)\hat{\mathbf{y}} + h(x,y)\hat{\mathbf{z}}$$
$$(x+y+z)\,(x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}})$$

9. The vector function below represents a possible electrostatic field:

$$E_x = 6xy \qquad E_y = 3x^2 - 3y^2 \qquad E_z = 0$$

(i) Calculate the line integral of $\vec{\mathbf{E}}$ from the point $(0,0,0)$ to the point $(x_1, y_1, 0)$ along the path which runs straight from $(0,0,0)$ to $(x_1, 0, 0)$ and then to $(x_1, y_1, 0)$.

(ii) Make a similar calculation for the path going first to $(0, y_1, 0)$. You should get the same answer if the function above is a true electrostatic field.

(iii) The line integral gives you a scalar potential function $\varphi(x, y, z)$. Take the gradient of this function and see that you get back the components of the given field.