UNIVERSITY OF ALABAMA
Department of Physics and Astronomy

PH 126 / LeClair Fall 2009

Problem Set 4: Circuits

Instructions:

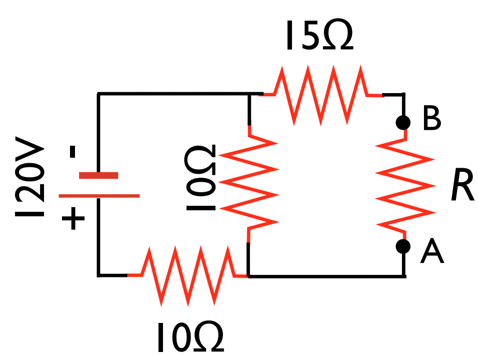
1. Answer all questions below.
2. Some problems have different due dates!
3. You may collaborate, but everyone must turn in their own work

The following problems are due Wed 23 September 2009

1. Two resistors are connected in parallel, with values R_1 and R_2. A total current I_o divides somehow between them. Show that the condition $I_1 + I_2 = I_o$, together with the requirement of minimum power dissipation, leads to the same current values that we would calculate with normal circuit formulas. This illustrates a general variational principle that holds for direct current networks: the distribution of currents within the networks, for a given input current I_o, is always that which gives the least total power dissipation.

2. Show that if a battery of fixed internal voltage ΔV and internal resistance r is connected to a variable external resistance R the maximum power is delivered to the external resistor when $r = R$.

3. A resistor R is to be connected across the terminals A, B of the circuit below. **(a)** For what value of R will the power dissipated in the resistor be the greatest? To answer this, construct the Thèvenin equivalent circuit and then invoke the result of the previous problem. **(b)** How much power will be dissipated in R?



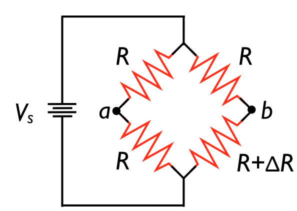
4. Two graphite rods are of equal length. One is a cylinder of radius a. The other is conical, tapering linearly from a radius a at one end to radius b at the other. Show that the end-to-end electrical resistance of the conical rod is a/b times that of the cylindrical rod. *Hint: consider the rod to be made up of thin, disk-like slices, all in series.*

The following problems are due Fri 25 September 2009

5. A laminated conductor was made by depositing, alternately, layers of silver 10 nm thick and layers of tin 20 nm thick. The composite material, considered on a larger scale, may be considered a homogeneous but anisotropic material with electrical conductivity $\sigma_{\perp}$ for currents perpendicular to the planes of the layers, and a different conductivity $\sigma_{||}$ for currents parallel to that plane. Given that the conductivity of silver is 7.2 times that of tin, find the ratio $\sigma_{\perp}/\sigma_{||}$.

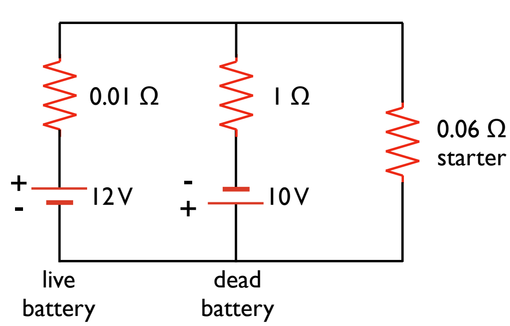
□ **6.** The circuit at right is known as a *Wheatstone Bridge*, and it is a useful circuit for measuring small changes in resistance. Perhaps you can figure out why. Three of the four branches on our bridge have identical resistance R, but the fourth has a slightly different resistance, ΔR more than the other branches, such that its total resistance is $R + \Delta R$.

In terms of the source voltage V_s, base resistance R and change in resistance ΔR, what is the potential difference between points a and b? You may assume the voltage source and wires are perfect (no internal resistance and no voltage drop, respectively).



Wheatstone Bridge

7. A dead battery is charged by connecting it to the live battery of another car with jumper cables (see below). Determine the current in the starter and in the dead battery.



8. A hair dryer intended for travelers operates at 115 V and also at 230 V. A switch on the dryer adjusts

the dryer for the voltage in use. At each voltage, the dryer delivers 1000 W of heat.

(a) What must the resistance of the heating coils be for each voltage?
(b) For such a dryer, sketch a circuit consisting of two identical heating coils connected to a switch and the power outlet. Opening and closing the switch should give the proper resistance for each voltage.
(c) What is the current in the heating elements at each voltage?

9. Two capacitors, one charged and the other uncharged, are connected in parallel. **(a)** Prove that when equilibrium is reached, each carries a fraction of the initial charge equal to the ratio of its capacitance to the sum of the two capacitances. **(b)** Show that the final energy is less than the initial energy, and derive a formula for the difference in terms of the initial charge and the two capacitances.