UNIVERSITY OF ALABAMA
Department of Physics and Astronomy

PH 126 / LeClair Fall 2009

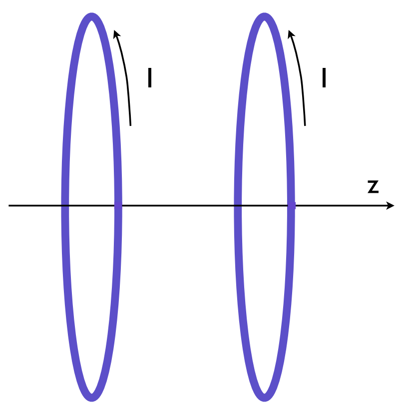
Problem Set 5: Magnetic Fields

Instructions:

1. Answer all questions below.
2. Some problems have different due dates!
3. You may collaborate, but everyone must turn in their own work

The following problems are due Wed 30 September 2009

1. A Helmholtz coil consists of two identical circular coils separated by a distance$d$, as shown at right. Each carries current I in the same direction. **(a)** Find the field at any point along the coil axis between the two coils. **(b)** Sow that $\partial B/\partial z$ vanishes at the midpoint. **(c)** Find d such that $\partial^2 B/\partial z^2$ also vanishes at the midpoint of the two coils.



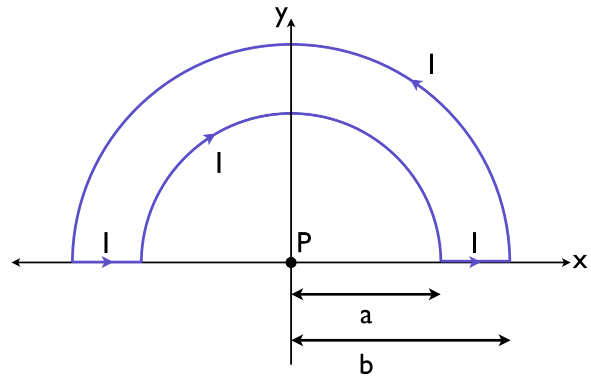
2. A current loop of radius a lies in the xy plane carrying a steady current I. Show that for distances large compared to a the resulting magnetic field may be written

$$B_r = \frac{\mu_0 |\vec{\mu}| \cos\theta}{2\pi r^3} \tag{1}$$

$$B_\theta = \frac{\mu_0 |\vec{\mu}| \sin\theta}{4\pi r^3} \tag{2}$$

Here r is the distance from the center of the loop, and θ is relative to the z axis (which is perpendicular to the plane of the loop).

3. Find the magnetic field at point P due to the current distribution shown below. *Hint: Break the loop into segments, and use superposition.*



4. Consider the magnetic field of a circular current ring, at points on the axis of the ring (use the exact formula, not your approximate form above). Calculate explicitly the line integral of the magnetic field along the ring axis from $-\infty$ to ∞, and check the general formula

$$\int \vec{B} \cdot d\vec{l} = \mu_o I_{encl} \tag{3}$$

Why may we ignore the "return" part of the path which would be necessary to complete a closed loop?

The following problems are due Fri 2 October 2009

5. The electric field of a long, straight line of charge with λ coulombs per meter is

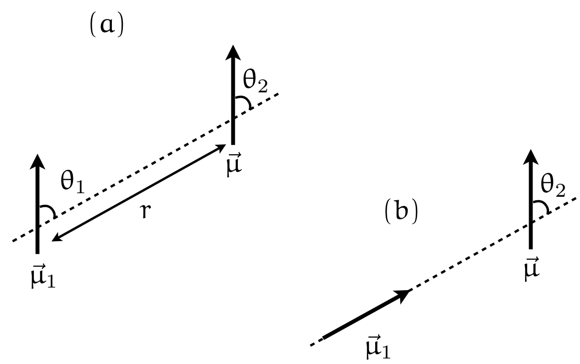
$$E = \frac{2k_e\lambda}{r}$$

where r is the distance from the wire. Suppose we move this line of charge parallel to itself at speed v. **(a)** The moving line of charge constitutes an electric current. What is the magnitude of this current? **(b)** What is the magnitude of the magnetic field produced by this current? **(c)** Show that the magnitude of the magnetic field is proportional to the magnitude of the electric field, and find the constant of proportionality.

6. A sphere of radius R carries the charge Q which is distributed uniformly over the surface of the sphere with a density $\sigma = 4\pi R^2$. This shell of charge is rotating about an axis of the sphere with angular velocity ω, in radians/sec. Find its magnetic moment. (Divide the sphere into narrow bands of rotating charge, find the current to which each band is equivalent and its dipole moment, and integrate over all bands.)

7. We want to find the energy required to bring two dipoles from infinite separation into the configuration shown in (a) below, defined by the distance r apart and the angles θ_1 and θ_2. Both dipoles lie in the plane of the paper. Perhaps the simplest way to compute the energy is this: bring the dipoles in from infinity while keeping them in the orientation shown in (b). This takes no work, for the force on each dipole is zero. Now calculate the work done in rotating $\vec{\mu}_1$ into its final orientation while holding $\vec{\mu}_2$ fixed. Then calculate the work required to rotate $\vec{\mu}_2$ into its final orientation. Thus show that the total work done, which we may call the potential energy of the system, is

$$U = \frac{\mu_1 \mu_2 \left(\sin\theta_1 \sin\theta_2 - 2\cos\theta_1 \cos\theta_2\right)}{r^3} \tag{4}$$



8. A metal crossbar of mass m slides without friction on two long parallel rails a distance b apart. A resistor R is connected across the rails at one end; compared with R, the resistance of the bar and rails is negligible. There is a uniform field $\vec{B}$ perpendicular to the plane of the figure. At time $t = 0$, the crossbar is given a velocity v_o toward the right. What happens then? **(a)** Does the rod ever stop moving? If so, when? **(b)** How far does it go? **(c)** How about conservation of energy? *Hint: first find the acceleration, and make use of an instantaneous balance of power.*

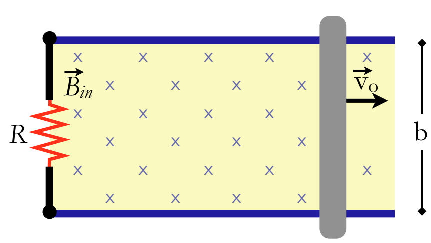


Figure 1: *Problem 5*