UNIVERSITY OF ALABAMA
Department of Physics and Astronomy

PH 126 / LeClair Fall 2009

# Problem Set 6: Relativity

**Instructions:**

1. Answer all questions below.
2. **All problems are due 16 October 2009**
3. You may collaborate, but everyone must turn in their own work

**1.** A flexible drive belt runs over two flywheels whose axles are mounted on a rigid base (Fig. 1). In the reference frame of the base, the horizontal portions of the belt have a speed $v$ and therefore are subject to length contraction, which tightens the belt around the flywheels. However, in a reference frame moving to the right with the upper portion of the belt, the *base* is subject to length contraction, which ought to loosen the belt around the flywheels. Resolve this "paradox" with by a qualitative argument. *Hint: consider the lower portion of the belt as seen in the reference frame of the upper portion.*

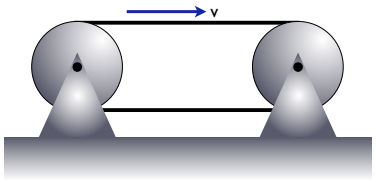


**Figure 1:** *Question 1*

**2.** Show that the velocity of a relativistic particle can be expressed as follows:

$$\vec{v} = \frac{c\,\vec{p}}{\sqrt{m^2c^2 + p^2}}$$

**3.** A car speeds past an observer on the ground at $0.9c$. A passenger in the car throws a ball out the car window at $0.7c$ relative to the car. What is the velocity of the ball with respect to the observer on the ground?

**4.** A cosmic-ray muon ($\mu$) is moving vertically through the atmosphere with a speed $v = 0.99c$. Its mean life expectancy against radioactive decay into an electron and two neutrinos is $2.22\ \mu s$, as measured in its own "rest" system. What will be its mean life expectancy as viewed by an observer on earth?

**5.** A stick of length $L$ is at rest on one system and is oriented at an angle $\theta$ with respect to the $x$ axis. What are the apparent length and orientation angle of this stick as viewed by an observer moving at a speed $v$ with respect to the first system?

**6.** A particle appears to move with speed $u$ at an angle $\theta$ with respect to the $x$ axis in a certain system. At what speed and angle will this particle appear to move in a second system moving with speed $v$ with respect to the first? Why does the answer differ from that of the previous problem?

**7.** The nonrelativistic expression for the momentum of a particle $p = mv$ agrees with experiments when $v \ll c$. For what speed does the nonrelativistic equation give an error of **(a)** $1.0\%$? **(b)** $5.0\%$?

**8.** An interstellar space probe is moving at a constant speed relative to earth of $0.76c$ toward a distant planet. Its radioisotope generators have enough energy to keep its data transmitter active continuously for $15$ years, as measured in their own reference frame. **(a)** How long do the generators last as measured from earth? **(b)** How far is the probe from earth when the generators fail, as measured from earth? **(c)** How far is the probe from earth when the generators fail, *as measured by its built-in trip odometer*?

**9.** The acceleration of a particle in one reference frame is $a_x = dv_x/dt$, where the particle has an instantaneous velocity $v_x$ in that frame. Consider a reference frame moving with speed $V$ parallel to the positive $x$ axis of the first frame. Show that the acceleration in the second frame is given by

$$a'_x = \frac{dv'_x}{dt} = a_x \frac{\left(1 - V^2/c^2\right)^{3/2}}{\left(1 - v_x V/c^2\right)^3}$$

**10.** A pion at rest ($m_\pi = 273\, m_{e^-}$) decays to a muon ($m_\mu = 207\, m_{e^-}$) and an antineutrino ($m_{\overline{\nu}} \approx 0$). This reaction is written as $\pi^- \rightarrow \mu^- + \overline{\nu}$. Find the kinetic energy of the muon and the energy of the antineutrino in electron volts. *Hint: relativistic momentum is conserved.*