UNIVERSITY OF ALABAMA
Department of Physics and Astronomy

PH 126 / LeClair Fall 2009

Problem Set 7: Induction, mostly

Instructions:

1. Answer all questions below.
2. **All problems are due 23 October 2009**
3. You may collaborate, but everyone must turn in their own work

1. A thin ring of radius a carries a static charge q. This ring is in a magnetic field of strength B_o, parallel to the ring's axis, and is supported so that it is free to rotate about that axis. If the field is switched off, how much angular momentum will be added to the ring? If the ring has mass m, show that it will acquire an angular velocity $\omega = qB_o/2mc$. *How is angular momentum being added? Consider the induced electric field and the resulting forces.*

2. An ocean current flows at a speed of 1.4 m/s in a region where the vertical component of the earth's magnetic field is 3.5×10^{-5} T. The resistivity of seawater in that region is about $\rho = 0.25\,\Omega\,\text{m}$. On the assumption that there is no other horizontal component of $\vec{E}$ other than the motional term $\vec{v} \times \vec{B}$, find the horizontal current density J in A/m^2? *NB – recall the general version of Ohm's law, viz.* $E = \rho J$. If you carried a bottle of seawater through the earth's field at this speed, would such a current be flowing in it?

3. A taut wire passes through the gap of a small magnet, where the field strength is 0.5 T. The length of the wire within the gap is 1.8 cm. Calculate the amplitude of the induced alternating voltage when the wire is vibrating at its fundamental frequency of 2000 Hz with an amplitude of 0.03 cm.

4. Derive an approximate formula for the mutual inductance of two circular rings of the same radius a, arranged like wheels on the same axis with their centers a distance b apart. Use an approximation good for $b \gg a$. *Recall that mutual inductance is the ratio of the flux through one object (say, ring 1) per unit current in the other object (say, ring 2).*

5. A conducting bar of length l rotates with a constant angular speed ω about a pivot at one end. A uniform magnetic field $\vec{B}$ is directed perpendicular to the plane of rotation, as shown in the figure below. Find the voltage induced between the ends of the bar.

6. A capacitor consists of two parallel rectangular plates with a vertical separation of 0.02 m. The east-west dimension of the plates is 0.2 m, the north-south dimension is 10 cm. The capacitor has been charged by connecting it temporarily to a battery of 300 V.

(a) How many excess electrons are on the negative plate?
(b) What is the electric field strength between the plates?

Now, give the quantities as they would be measured in a frame of reference which is moving eastward, relative to the laboratory in which the plates are at rest, with speed $0.6c$.

(c) The dimensions of the capacitor,
(d) The number of excess electrons on the negative plate,
(e) The electric field strength between the plates.

Now answer the same questions above (c-e) for a frame of reference which is moving upward with speed $0.6c$.

7. Consider the trajectory of a charged particle which is moving with speed $0.8c$ in the x direction when it enters a large region in which there is a uniform electric field in the y direction. Show that the x velocity of the particle must actually *decrease*. What about the x component of momentum?

8. Very large magnetic fields can be produced using a procedure called *flux compression.* A metallic cylindrical tube of radius R is placed coaxially in a long solenoid of somewhat larger radius. The space between the tube and the solenoid is filled with a highly explosive material. When the explosive is set off, it collapses the tube to a cylinder of radius $r < R$. If the collapse happens very rapidly, induced current in the tube maintains the magnetic flux nearly constant inside the tube, even though the area shrinks. If the initial magnetic field in the solenoid is $2.50\,\mathrm{T}$, and $R/r = 12.0$, what is the maximum field that can be reached?