

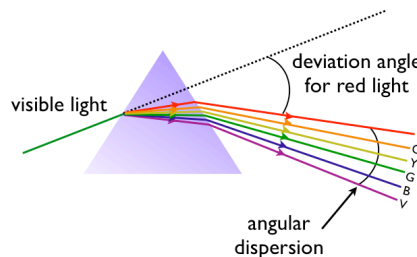
Problem Set 8: Geometric optics, mostly

Instructions:

1. Answer all questions below.
2. **All problems are due 13 October 2009**
3. You may collaborate, but everyone must turn in their own work

1. The index of refraction for **violet** light in silica flint glass is $n_{\text{violet}} = 1.66$, and for **red** light it is $n_{\text{red}} = 1.62$. In air, $n = 1$ for both colors of light.

What is the **angular dispersion** of visible light (the angle between red and violet) passing through an equilateral triangle prism of silica flint glass, if the angle of incidence is 50° ? The angle of incidence is that between the ray and a line *perpendicular* to the surface of the prism. Recall that all angles in an equilateral triangle are 60° .



2. As light from the Sun enters the atmosphere, it refracts due to the small difference between the speeds of light in air and in vacuum. The optical length of the day is defined as the time interval between the instant when the top of the Sun is just visibly observed above the horizon, to the instant at which the top of the Sun just disappears below the horizon. The geometric length of the day is defined as the time interval between the instant when a geometric straight line drawn from the observer to the top of the Sun just clears the horizon, to the instant at which this line just dips below the horizon. The day's optical length is slightly larger than its geometric length.

By how much does the duration of an optical day exceed that of a geometric day? Model the Earth's atmosphere as uniform, with index of refraction $n = 1.000293$, a sharply defined upper surface, and depth 8767 m. Assume that the observer is at the Earth's equator so that the apparent path of the rising and setting Sun is perpendicular to the horizon. You may take the radius of the earth to be 6.378×10^6 m. Express your answer to the nearest hundredth of a second.

3. What is the apparent depth of a swimming pool in which there is water of depth 3 m, (a) When viewed from normal incidence? (b) When viewed at an angle of 60° with respect to the surface? The refractive index of water is 1.33.

4. A conducting rectangular loop of mass M , resistance R , and dimensions w by l falls from rest into a magnetic field \vec{B} , as shown at right. At some point before the top edge of the loop reaches the magnetic field, the loop attains a constant terminal velocity v_T . Show that the terminal velocity is:

$$v_T = \frac{MgR}{B^2w^2}$$

5. A point source of light is placed at a fixed distance l from a screen. A thin convex lens of focal length f is placed somewhere between the source and screen, a distance q from the screen and p from the source. The lens is moved back and forth between the source and screen, but both screen and source remain fixed, thus $p + q = l$ at all times.

What is the minimum value of l such that a focused image will be formed at two different positions of the lens? Recall our recent laboratory experiment.

6. Consider two solenoids, one of which is a tenth-scale model of the other. The larger solenoid is 2 m long, and 1 m in diameter, and is wound with 1 cm-diameter copper wire. When the coil is connected to a 120 V dc generator, the magnetic field at the center is exactly 0.1 T. The scaled-down version is exactly one-tenth the size in every linear dimension, including the diameter of the wire. The number of turns is the same in both coils, and both are designed to provide the same central field.

(a) Show that the voltage required is the same, namely, 120 V

(b) Compare the coils with respect to the power dissipated, and the difficulty of removing this heat by some cooling means.

7. The walls of a prison cell are perpendicular to the four cardinal compass directions. On the first day of spring, light from the rising Sun enters a rectangular window in the eastern wall. The light traverses 2.57 m horizontally to shine perpendicularly on the wall opposite the window. A prisoner observes the patch of light moving across this western wall and for the first time forms his own understanding of the rotation of the Earth. (a) With what speed does the illuminated rectangle move? (b) The prisoner holds a small square mirror flat against the wall at one corner of the rectangle of light. The mirror reflects light back to a spot on the eastern wall close beside the window. How fast does the smaller square of light move across that wall?