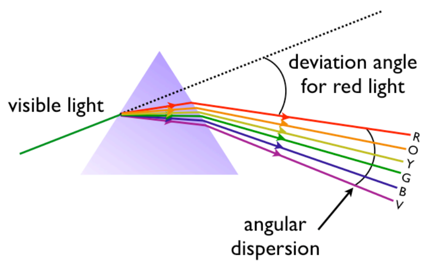
University of Alabama
Department of Physics and Astronomy

PH 126 / LeClair Fall 2009

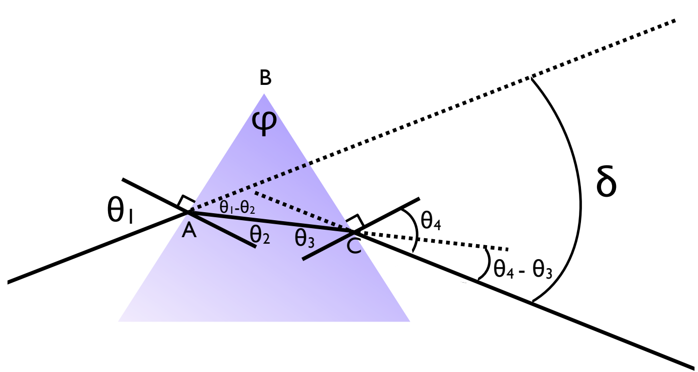
Problem Set 8 solutions

1. *Serway 35.35* The index of refraction for **violet** light in silica flint glass is $n_{\text{violet}} = 1.66$, and for **red** light it is $n_{\text{red}} = 1.62$. In air, $n = 1$ for both colors of light.

What is the **angular dispersion** of visible light (the angle between red and violet) passing through an equilateral triangle prism of silica flint glass, if the angle of incidence is $50°$? The angle of incidence is that between the ray and a line *perpendicular* to the surface of the prism. Recall that all angles in an equilateral triangle are $60°$.



What we need to do is find the deviation angle for both red and violet light in terms of the incident angle and refractive index of the prism. The angular dispersion is just the difference between the deviation angles for the two colors. First, let us define some of the geometry a bit better, referring to the figure below.



Let the angle of incidence be θ_1, and the refracted angle θ_2 at point A. The incident and refracted angles are defined with respect to a line *perpendicular* to the prism's surface. Similarly, when the light rays exit the prism, we will call the incident angle within the prism θ_3, and the refracted angle exiting the prism θ_4 at point C. If we call index of refraction of the prism n, and presume the surrounding material is just air with index of refraction 1.00, we can apply Snell's law at both interfaces:

$$n \sin\theta_2 = \sin\theta_1$$
$$n \sin\theta_3 = \sin\theta_4$$

Fair enough, but now we need to use some geometry to relate these four angles to each other, the deviation angle δ, and the prism's apex angle φ. Have a look at the triangle formed by points A, B, and C. All three angles in this triangle must add up to $180°$. At point A, the angle between the prism face and the line $\overline{AC}$ is $\angle BAC = 90 - \theta_2$ - the line we drew to define θ_1 and θ_2 is by construction perpendicular to the prism's face, and thus makes a $90°$ angle with respect to the face. The angle $\angle BAC$ is all of that $90°$ angle, *minus* the refracted angle θ_2. Similarly, we can find $\angle BCA$ at point C. We know the apex angle of the prism is φ, and for an equilateral triangle, we must have $\varphi = 60°$

$$(90° - \theta_2) + (90° - \theta_3) + \varphi = 180°$$
$$\implies \varphi = \theta_2 + \theta_3 = 60°$$

How do we find the deviation angle? Physically, the deviation angle is just how much in total the exit ray is "bent" relative to the incident ray. At the first interface, point A, the incident ray and reflected ray differ by an angle $\theta_1 - \theta_2$. At the second interface, point C, the ray inside the prism and the exit ray differ by an angle $\theta_4 - \theta_3$. These two differences *together* make up the total deviation - the deviation is nothing more than adding together the differences in angles at each interface due to refraction. Thus:

$$\delta = (\theta_1 - \theta_2) + (\theta_4 - \theta_3) = \theta_1 + \theta_4 - (\theta_2 + \theta_3)$$

Of course, one can prove this rigorously with quite a bit more geometry, but there is no need: we know physically what the deviation angle is, and can translate that to a nice mathematical formula. Now we can use the expression for φ in our last equation:

$$\delta = \theta_1 + \theta_4 - \varphi$$

We were given $\theta_1 = 50°$, so now we really just need to find θ_4 and we are done. From Snell's law above, we can relate θ_4 to θ_3 easily. We can also relate θ_3 to θ_2 and the apex angle of the prism, φ. Finally, we can relate θ_2 back to θ_1 with Snell's law. First, let us write down all the separate relations:

$$\begin{aligned} \sin\theta_4 &= n\sin\theta_3 \\ \theta_3 &= \varphi - \theta_2 \\ n\sin\theta_2 &= \sin\theta_1 \\ \text{or}\quad \theta_2 &= \sin^{-1}\left(\frac{\sin\theta_1}{n}\right) \end{aligned}$$

If we put all these together (in the right order) we have θ_4 in terms of known quantities:

$$\begin{aligned} \sin\theta_4 &= n\sin\theta_3 \\ &= n\sin(\varphi - \theta_2) \\ &= n\sin\left[\varphi - \sin^{-1}\left(\frac{\sin\theta_1}{n}\right)\right] \end{aligned}$$

With that, we can write the full expression for the deviation angle:

$$\delta = \theta_1 + \theta_4 - \varphi = \theta_1 + n\sin\left[\varphi - \sin^{-1}\left(\frac{\sin\theta_1}{n}\right)\right] - \varphi$$

Now we just need to calculate the deviation separately for red and violet light, using their different indices of refraction. You should find:

$$\begin{aligned} \delta_{\text{red}} &= 48.56° \\ \delta_{\text{blue}} &= 53.17° \end{aligned}$$

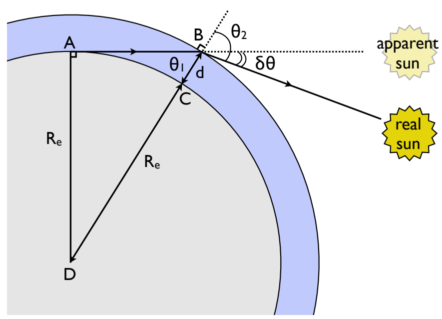
The angular dispersion is just the difference between these two:

$$\text{angular dispersion} = \delta_{\text{blue}} - \delta_{\text{red}} = 4.62°$$

2. *Serway 35.62* As light from the Sun enters the atmosphere, it refracts due to the small difference between the speeds of light in air and in vacuum. The optical length of the day is defined as the time interval between the instant when the top of the Sun is just visibly observed above the horizon, to the instant at which the top of the Sun just disappears below the horizon. The geometric length of the day is defined as the time interval between the instant when a geometric straight line drawn from the observer to the top of the Sun just clears the horizon, to the instant at which this line just dips below the horizon. The day's optical length is slightly larger than its geometric length.

By how much does the duration of an optical day exceed that of a geometric day? Model the Earth's atmosphere as uniform, with index of refraction $n = 1.000293$, a sharply defined upper surface, and depth 8767 m. Assume that the observer is at the Earth's equator so that the apparent path of the rising and setting Sun is perpendicular to the horizon. You may take the radius of the earth to be 6.378×10^6 m. Express your answer to the nearest hundredth of a second.

First, we need to draw a little picture. This is the situation we have been given:



We presume that some human is standing at point A on the earth's surface, looking straight out toward the horizon. This line of sight intersects the boundary between the atmosphere and space (which we are told to assume is a sharp one) at point B. Light rays from the sun, which is slightly below the horizon, are refracted toward the earth's surface at point B, and continue on along the line of sight from B to A. We know the index of refraction of vacuum is just unity ($n_{vacuum} = 1$), while that of the atmosphere is $n = 1.000293$. The day appears to be slightly longer because we see the sun even after it has gone through an extra angle of rotation $\delta\theta$ due to atmospheric refraction.

To set up the geometry, we first draw a radial line from point B to the center of the earth. This line, $\overline{BC}$, will intersect the boundary of the atmosphere at point B, and will be normal to the atmospheric boundary. This defines the angle of incidence θ_2 and the angle of refraction θ_1 for light coming from the sun. The difference between these two angles, $\delta\theta$, is how much the light is bent downward upon being refracted from the atmosphere. How do we relate this to the extra length of the day one would observe? We know that the earth revolves on its axis at a constant angular speed - one revolution in 24 hours. Thus, we can easily find the angular speed of the earth:

$$\text{earth's angular speed} = \omega = \frac{\text{one revolution}}{1\text{day}} = \frac{360°}{86400\,\text{s}}$$

Here we used the fact that there are $24 \cdot 60 \cdot 60 = 86400$ seconds in one day. Given the angular velocity of the earth, we know exactly how long it will take for the earth to rotate through the "extra" angle $\delta\theta$ due to refraction:

$$\delta\theta = \omega\delta t$$

We only need one last bit: the atmospheric refraction occurs *twice per day* – once at sun-up and once at sun-down. The total "extra" length of the day is then $2\delta t$. Thus, if we can find $\delta\theta$, we can figure out how much longer the day seems to be due to atmospheric refraction. In order to find it, we need to use the law of refraction and a bit of geometry. First, from the law of refraction and the fact that $\delta\theta = \theta_2 - \theta_1$, we can state the following:

$$\theta_2 - \theta_1 = \delta\theta$$
$$n \sin\theta_1 = \sin\theta_2 = \sin(\theta_1 + \delta\theta)$$

In order to proceed further, we draw a line from point A to the center of the earth, point D. This forms a triangle, $\triangle ABD$. Because line $\overline{AD}$ is a radius of the earth, by construction, it must intersect line $\overline{AB}$ at a right angle, since the latter is by construction a tangent to the earth's surface. Thus, $\triangle ABD$ is a right triangle, and

$$\sin\theta_1 = \frac{\overline{AD}}{\overline{BD}} = \frac{R_e}{R_e + d}$$

Plugging this into the previous equation,

$$n \sin\theta_1 = \sin\theta_2 = \sin(\theta_1 + \delta\theta) = n\frac{R_e}{R_e + d}$$

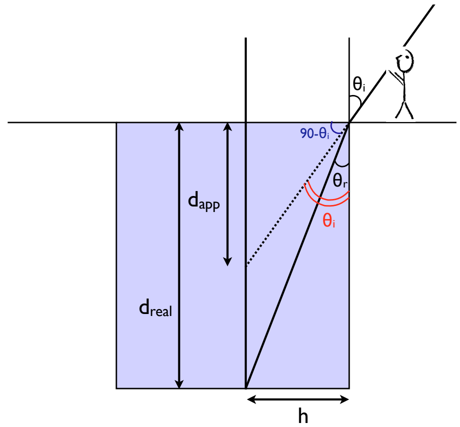
In principle, we are done at this point. The previous expression allows one to calculate θ_1, while the present one allows one to find $\delta\theta$ if θ_1 is known. From that, one only needs the angular speed of the earth.

$$\theta_2 = \theta_1 + \delta\theta = \sin^{-1}\left[\frac{nR_e}{R_e + d}\right]$$
$$\delta\theta = \sin^{-1}\left[\frac{nR_e}{R_e + d}\right] - \theta_1 = \sin^{-1}\left[\frac{nR_e}{R_e + d}\right] - \sin^{-1}\left[\frac{R_e}{R_e + d}\right] = \omega\delta t$$
$$2\delta t = \frac{2\delta\theta}{\omega} \approx 163.82\,\mathrm{s}$$

Of course, it is more satisfying to have an analytic approximation. We will leave that as an exercise to the reader for now.

3. *Frank 16.1* What is the apparent depth of a swimming pool in which there is water of depth 3 m, **(a)** When viewed from normal incidence? **(b)** When viewed at an angle of $60°$ with respect to the surface? The refractive index of water is 1.33.

As always, we first need to draw a little picture of the situation at hand.



It is slightly more convenient to redefine the angle of incidence θ_i to be with respect to the *normal* of the water's surface itself, rather than with respect to the surface, since that is our usual convention. That means we are interested in incident angles for the observer of $90°$ and $30°$. The depth of the pool will be $d_{real} = 3\,\text{m}$. If an observer views the bottom of the pool with an angle θ_i with respect to the surface normal, refracted rays from the bottom of the pool will be bent away from the surface normal on the way to their eyes. That is, rays emanating from the bottom of the pool will make an angle $\theta_r < \theta_i$ with respect to the surface normal, and rays exiting the pool will make an angle θ_i with the surface normal. This is owing to the fact that the light will be bent *toward* the normal in the faster medium, the air, on exiting the water.

What depth does the observer actually see? They see what light would do in the absence of refraction, the path that light rays would appear to take if the rays were not "bent" by the water. In this case, that means that the observer standing next to the pool would think they saw the light rays coming from an angle θ_i with respect to the surface normal (dotted line in the pool). The *lateral* position of the bottom of the pool would remain unchanged. If the real light rays intersect the bottom of the pool a distance h from the edge, then the apparent bottom of the pool is also a distance h from the edge of the pool. Try demonstrating this with a drinking straw in a glass of water!

So what to do? First off, we can apply Snell's law. If the index of refraction of air is 1, and the water has an index of refraction n, then

$$n \sin\theta_r = \sin\theta_i$$

We can also use the triangle defined by d_{real} and h:

$$\tan\theta_r = \frac{h}{d_{real}}$$

as well as the triangle defined by d_{real} and h[i]:

$$\tan(90-\theta_i) = \frac{d_{app}}{h} = \frac{1}{\tan\theta_i}$$

Solving the last two equations for h,

$$\begin{aligned} h &= d_{real}\tan\theta_r = d_{app}\tan\theta_i \\ \implies \quad d_{app} &= d_{real}\left[\frac{\tan\theta_r}{\tan\theta_i}\right] \end{aligned}$$

From Snell's law, we have a relationship between θ_r and θ_i already:

$$\theta_r = \sin^{-1}\left[\frac{\sin\theta_i}{n}\right]$$

Putting everything together,

$$d_{app} = \frac{d_{real}}{\tan\theta_i}\tan\theta_r = \frac{d_{real}}{\tan\theta_i}\left[\tan\left(\sin^{-1}\left[\frac{\sin\theta_i}{n}\right]\right)\right]$$

If you just plug in the numbers at this point, you have a problem. One of the angles is $\theta_i = 0$, normal incidence, which means we have to divide by zero in the expression above. Dividing by zero is worse than drowning kittens, far worse. Thankfully, we know enough trigonometry to save the poor kittens.

We can save the kittens by remembering an identity for $\tan\left[\sin^{-1}x\right]$. If we have an equation like $y = \sin^{-1}x$, it implies $\sin y = x$. This means y is an angle whose sine is x. If y is an angle in a right triangle, then it has an opposite side x and a hypotenuse 1, making the adjacent side $\sqrt{1-x^2}$. The tangent of angle y must then be $x/\sqrt{1-x^2}$. Thus,

$$\tan\left[\sin^{-1}x\right] = \frac{x}{\sqrt{1-x^2}}$$

Using this identity in our equation for d_{app},

$$d_{app} = \frac{d_{real}}{\tan\theta_i}\left[\frac{\sin\theta_i}{n\sqrt{1-\left[\frac{\sin\theta_i}{n}\right]^2}}\right] = \frac{d_{real}}{\tan\theta_i}\left[\frac{\sin\theta_i}{\sqrt{n^2-\sin^2\theta_i}}\right] = \frac{d_{real}\cos\theta_i}{\sqrt{n^2-\sin^2\theta_i}}$$

[i] Along with an identify for $\tan\theta$, *viz.*, $\tan(90-\theta) = 1/\tan\theta$

Viewed from normal incidence with respect to the surface means $\theta_i = 0$ – looking straight down at the surface of the water. In this case, $\sin\theta_i = 0$, and the result is simple:

$$d_{app} = \frac{d_{real}}{n} \approx 2.6\,\text{m}$$

Viewed from 60° with respect to the *surface* means 30° with respect to the normal, and thus

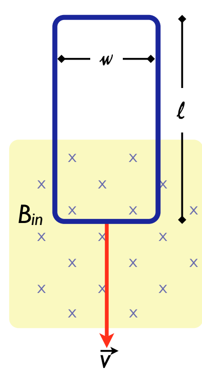
$$d_{app} = d_{real}\cos 30\left[\frac{1}{\sqrt{1.33^2 - \sin^2 30}}\right] \approx 2.1\,\text{m}$$

There are easier ways to solve the normal incidence problem, without endangering any kittens whatsoever. Solving that problem, however, is a special case, and of limited utility. You would still have to solve the case of 60° incidence separately. I wanted to show you here that solving the general problem just once is all you need to do, so long as you are careful enough.

4. A conducting rectangular loop of mass M, resistance R, and dimensions w by l falls from rest into a magnetic field $\vec{B}$, as shown at right. At some point before the top edge of the loop reaches the magnetic field, the loop attains a constant terminal velocity v_T. Show that the terminal velocity is:

$$v_T = \frac{MgR}{B^2 w^2}$$

NB – terminal velocity is reached when the net acceleration is zero. See the schematic figure on the next page.



First, let us analyze the situation qualitatively. As the loop falls into the region of magnetic field, more of its area is exposed to the field, which increases the total flux through the loop. This increase in magnetic flux will cause an induced potential difference around the loop, via Faraday's law, which will create a current that tries to counteract this change in magnetic flux. Since the flux is increasing, the induced current in the loop will try to act *against* the existing field to reduce the change in flux, which means the current will circulate counterclockwise to create a field out of the page.

Once there is a current flowing in the loop, each current-carrying segment will feel a magnetic force. The left and right segments of the loop will have equal and opposite forces, leading to no net effect, but the current flowing (to the right) in the bottom segment will lead to a force $F_B = BIw$ upward. Again, this is consistent with Faraday's (and Lenz's) law - any magnetic force on the loop must act in such a way to reduce the rate at which the flux changes, which in this case clearly means slowing down the loop. The upward force on the loop will serve to counteract the gravitational force, which is ultimately responsible for the flux change in this case anyway. The faster the loop falls, the larger the upward force it experiences, and at some point the magnetic force will balance the gravitational force perfectly, leading to no net acceleration, and hence constant velocity. This is the "terminal velocity." Of course, once the whole loop is inside the magnetic field, the flux is again constant, and the loop just starts to fall normally again.[ii]

Quantitatively, we must first find the induced voltage around the loop, which will give us the current. The current will give us the force, which will finally give us the acceleration. As the loop falls into the magnetic field, at some instant t we will say that a length x of the loop has moved into the field, out of the total length l. At this time, the total flux through the loop is then:

$$\Phi_B = \vec{B} \cdot \vec{A} = BA = Bwx$$

From the flux, we can easily find the induced voltage from Faraday's law.

$$\Delta V = -\frac{\Delta \Phi_B}{\Delta t} = -Bw\frac{\Delta x}{\Delta t} = -Bwv$$

Here we made use of the fact that the rate at which the length of the loop exposed to the magnetic field changes is simply the instantaneous velocity, $\Delta x / \Delta t = v$. Once we have the induced voltage, given the resistance of the loop R, we know the current via Ohm's law:

$$I = \frac{\Delta V}{R} = -\frac{Bwv}{R}$$

From Lenz's law we know the current circulates counterclockwise. In the right-most segment of the loop, the current is flowing up, and the magnetic field into the page. The right-hand rule then dictates that the force on this current-carrying segment must be to the left. The left-most segment of the loop has a force equal in magnitude, since the current I, the length of wire, and the magnetic field are the same, but the force is in the opposite direction. Thus, taken together, the left and right segments of the loop

[ii] We would still have eddy currents, which would provide some retarding force, but for thin wires eddy current forces are probably going to be negligible. This is basically what we demonstrated with our conducting pendulums swinging through a magnetic field. The pendulums that had only thin segments of conductor (it looked like a fork) experienced very little damping compared to a plain flat plate.

contribute no net force. The bottom segment, however, experiences an upward force, since the current is to the right. For a constant magnetic field and constant current (true at least instantaneously), the force is easily found:

$$F_B = BIw$$

We can substitute our expression for I above:

$$F_B = BIw = -\frac{B^2w^2v}{R}$$

At the terminal velocity v_T, this upward force will exactly balance the downward gravitational force:

$$\begin{aligned} & \sum F = mg - \frac{B^2w^2v_T}{R} = 0 \\ \Longrightarrow \quad & v_T = \frac{mgR}{B^2w^2} \end{aligned}$$

5. A point source of light is placed at a fixed distance l from a screen. A thin convex lens of focal length f is placed somewhere between the source and screen, a distance q from the screen and p from the source. The lens is moved back and forth between the source and screen, but both screen and source remain fixed, thus $p + q = l$ at all times.

What is the minimum value of l such that a focused image will be formed at two different positions of the lens? Recall our recent laboratory experiment.

What we are basically told is that $p + q = l$ at all times. We can use this along with the lens equation to come up with a set of solutions for q in terms of l and f - we will get a quadratic, and we will be able to readily see what conditions give two, one, or no real solutions.

$$\begin{aligned}
l &= p + q \\
\frac{1}{f} &= \frac{1}{p} + \frac{1}{q} \\
\implies \quad \frac{1}{q} &= \frac{1}{f} - \frac{1}{p} \\
\frac{1}{q} &= \frac{1}{f} - \frac{1}{l-q} \\
\frac{1}{q} &= \frac{l-q}{f(l-q)} - \frac{f}{f(l-q)} \\
\frac{1}{q} &= \frac{l-q-f}{f(l-q)}
\end{aligned}$$

Now we have an equation purely in terms of l, q, and f, which we can readily solve for q. Start by cross-mulitplying.

$$\begin{aligned}
f(l-q) &= q(l-q-f) \\
fl - fq &= ql - q^2 - qf \\
q^2 - lq + fl &= 0 \\
\implies \quad q &= \frac{l \pm \sqrt{l^2 - 4fl}}{2}
\end{aligned}$$

From the solution to the quadratic above, we can see that there are two real image positions when the factor under the square root is positive, when $l^2 > 4fl$ or $l > 4f$. When the length l is exactly four times the focal length, $l = 4f$, there is only one solution to the quadratic. Thus, the critical position is when $l = 4f$, which results in $q = \frac{l}{2} = p$.

6. Consider two solenoids, one of which is a tenth-scale model of the other. The larger solenoid is 2 m long, and 1 m in diameter, and is wound with 1 cm-diameter copper wire. When the coil is connected to a 120 V dc generator, the magnetic field at the center is exactly 0.1 T. The scaled-down version is exactly one-tenth the size in every linear dimension, including the diameter of the wire. The number of turns is the same in both coils, and both are designed to provide the same central field.

(a) Show that the voltage required is the same, namely, 120 V
(b) Compare the coils with respect to the power dissipated, and the difficulty of removing this heat by some cooling means.

This is basically a scaling problem: when everything is shrunk by 10 times, what happens to the required voltage for a given field? First, let's consider the large solenoid. Let's say it has length $L = 2$ m, radius $r = 0.5$ m, contains N turns of wire, and it provides a field $B = 0.1$ T with a current I. We know we can relate the field and the current:

$$B = \mu_0 \frac{N}{L} I$$

The solenoid is just a long single strand of wire wrapped around a cylinder. If we say that the total length of wire used to wrap the solenoid is l, and the wire's diameter is d, then we can calculate the resistance of the solenoid:

$$R = \frac{\rho l}{A} = \frac{\rho l}{\pi d^2/4}$$

Here we have used the wire's resistivity ρ, and its cross-sectional area $A = \pi r^2 = \pi d^2/4$. Given the resistance and voltage of $\Delta V = 120\,\text{V}$, we can calculate the current:

$$I = \frac{\Delta V}{R} = \frac{\Delta V \pi d^2/4}{\rho l}$$

Now if we plug that into our first solenoid equation above, we can relate voltage and magnetic field:

$$B = \mu_0 \frac{N}{L} I = \mu_0 \frac{N}{L} \frac{\Delta V \pi d^2/4}{\rho l} = \frac{\mu_0 \pi}{4\rho} \frac{N \Delta V d^2}{L l}$$

Now, what about the small solenoid? Every dimension is a factor of 10 smaller. If *all* the dimensions are 10 times smaller, the number of turns that fit within 1/10 the length is the *same* as the big solenoid if the wire diameter is also 1/10 as large! In other words, both coils will have the same number of turns - the space for the wire is 10 times smaller, but so is the wire.

In order to find the relationship for the small solenoid, we will use the same symbols, but everything for the small solenoid will have a prime $'$. The number of turns in the small solenoid is N', and in for the large solenoid it is just N. The voltage on the little solenoid is $\Delta V'$, and on the large one we have just ΔV. Using the results from above, magnetic field for the small solenoid is then easily found by substitution:

$$B' = \frac{\mu_0 \pi}{4\rho} \frac{N' \Delta V (d')^2}{L' l'} = B$$

We don't have to bother with a prime on the resistivity, both coils have the same sort of wire. Remember, our desired condition is that $B' = B$. We know that $N' = N$, and all the dimensions are 10 times smaller - the length of the solenoid, the wire diameter, and therefore also the length of wire required. We have the same number of *turns* in each coil, but in the smaller coil the circumference of each turn is 10 times smaller, which means overall, the total length of wire required l is 10 times smaller. Thus:

$$
\begin{aligned}
B' &= \frac{\mu_0\pi}{4\rho}\frac{N'\Delta V'(d')^2}{L'l'} \\
&= \frac{\mu_0\pi}{4\rho}\frac{N\Delta V'(d')^2}{L'l'} && \text{note that } N' = N \\
&= \frac{\mu_0\pi}{4\rho}\frac{N\Delta V'(\frac{d}{10})^2}{\frac{L}{10}\frac{l}{10}} && \text{scale all dimensions by } \frac{1}{10} \\
&= \frac{\mu_0\pi}{4\rho}\frac{N\Delta V' d^2}{Ll}
\end{aligned}
$$

Now, we want to enforce the condition that the field is the same in both solenoids:

$$
\begin{aligned}
B' &= B \\
\implies \quad \frac{\mu_0\pi}{4\rho}\frac{N\Delta V' d^2}{Ll} &= \frac{\mu_0\pi}{4\rho}\frac{N\Delta V d^2}{Ll} \\
\implies \quad \Delta V' &= \Delta V
\end{aligned}
$$

Thus, a solenoid shrunk by 10 times in every dimension will require the same applied voltage for the same magnetic field. What about the power consumption? The current in the large solenoid was

$$
I = \frac{\Delta V}{R} = \frac{\Delta V \pi d^2/4}{\rho l}
$$

In the small solenoid, we now know that the voltage is the same, but the resistance is not, so we should have:

$$
I' = \frac{\Delta V}{R'} = \frac{\Delta V \pi (d')^2/4}{\rho l'} = \frac{\Delta V \pi (\frac{d}{10})^2/4}{\rho \frac{l}{10}} = \frac{1}{10}\frac{\Delta V \pi d^2/4}{\rho l} = \frac{1}{10} I
$$

The current in the little solenoid is 10 times less - sensible, since the total length of wire is 10 times smaller, but the area of the wire is 100 times smaller. The power required for each is the product of current and voltage:

$$
\begin{aligned}
\mathscr{P}_{\text{big}} &= I\Delta V \\
\mathscr{P}_{\text{small}} &= I'\Delta V = \frac{1}{10} I\Delta V = \frac{1}{10}\mathscr{P}_{\text{big}}
\end{aligned}
$$

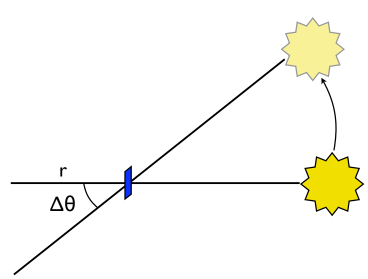
Not only is the larger solenoid ten times larger, it requires ten times more power, and therefore dissipates ten times more heat. The cooling requirements will be far more formidable for the larger solenoid. For instance, if we decide to use water cooling, the flow rate will need to be at least 10 times larger for the large solenoid to extract a heat load ten times larger. Not to mention the fact that we have to acquire a much larger power supply in the first place - practically speaking, the difference between a 5 A current source and a 50 A current source is significant. Keep in mind that your normal household outlets deliver 120 V at a maximum of ~15 A.

7. The walls of a prison cell are perpendicular to the four cardinal compass directions. On the first day of spring, light from the rising Sun enters a rectangular window in the eastern wall. The light traverses 2.57 m horizontally to shine perpendicularly on the wall opposite the window. A prisoner observes the patch of light moving across this western wall and for the first time forms his own understanding of the rotation of the Earth. **(a)** With what speed does the illuminated rectangle move? **(b)** The prisoner holds a small square mirror flat against the wall at one corner of the rectangle of light. The mirror reflects light back to a spot on the eastern wall close beside the window. How fast does the smaller square of light move across that wall?

The sun appears to move at an angular velocity ω, which means that it moves through an angular displacement $\Delta\theta$ in a time Δt: $\omega = \Delta\theta/\Delta t$. We know the rotation rate of the sun: it goes through a full circle of 2π radians in 24 hours:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \,\text{rad}}{86400\,\text{s}} \approx 7.27 \times 10^{-5}\,\text{rad/s}$$

The light streaming through the prison window will move through an angle $\Delta\theta$ as shown below as the sun moves through the sky:



If the distance the light covers along the wall is s, then it is clear that $s = \Delta\theta r$. The rate at which the spot moves is ds/dt. Since $r = 2.37$ m is constant,

$$\frac{ds}{dt} = \frac{d}{dt}\left(\Delta\theta r\right) = r\frac{d\Delta\theta}{dt} = r\omega = (2.37\,\text{m})\left(7.27 \times 10^{-5}\,\text{rad/s}\right) \approx 0.172\,\text{mm/s}$$

If the prisoner uses a mirror, the path length of the light is simply doubled, as if the room were twice as wide, so a given angular displacement $\Delta\theta$ results in twice as large a lateral displacement s, and twice the apparent speed $\mathrm{d}s/\mathrm{d}t$. Thus, for the second case, we have just 0.345 mm/s.