UNIVERSITY OF ALABAMA
Department of Physics and Astronomy

PH 106-4 / LeClair Fall 2008

Problem Set 7: Hints

1. As one of you put it, "Why would turning a static field OFF impart angular velocity?" The key is that you have a time-varying magnetic flux. Whether it increases or decreases is just a difference in sign, the important point is that the magnetic flux *changes*. This leads to an induced electric field, which gives an electric force, which imparts a torque. Torque nothing more than the rate of change of angular momentum.

Once the field is turned off, the flux in the loop changes from $\Phi_B = B_o \pi a^2$ to zero. This means that there must be an induced voltage around the ring,

$$\frac{\partial \Phi_B}{\partial t} = -\Delta V = \oint_{\text{ring}} \vec{E} \cdot d\vec{l}$$

Thinking about it another way, since $\vec{B}$ points parallel to the ring's axis, the rate of change of $\vec{B}$ also points parallel to the ring's axis. If $d\vec{B}/dt$ points along the ring axis, then the indued electric field must *circulate* around the axis, which we can see from

$$\frac{\partial B}{\partial t} = \vec{\nabla} \times \vec{E} \tag{1}$$

If there is an $\vec{E}$ field circulating around the perimeter of the ring, that means that each little bit of charge along the ring experiences a tangential force:

$$d\vec{F}_e = dq\,\vec{E} = \lambda\, dl E\,\hat{\theta}$$

Around the whole ring, there is no net force, since all the tangential bits around the circle cancel each other. However, since they all circulate in the same direction around the loop, there is a torque. The force applied on a bit dq is given above; the torque is just the radial distance from the center of the loop to the loop perimeter crossed into the force:

$$d\vec{\tau} = \vec{r} \times d\vec{F}_e \tag{2}$$

Integrate the $d\vec{\tau}$ around the perimeter of the loop, and you have the whole torque on the loop. Once you've done that, if you recall the right-hand side first equation above ... your integral for torque should yield the induced voltage. That allows you to relate $\vec{\tau}$ to $d\Phi_B/dt$. Noting that $\tau = dL/dt$, you can relate the time rate of change of angular momentum to the time rate of change of magnetic flux. Integrate both between initial ($L=0$ at $t=0$) and final (L_f at t) states, and you have L. For a solid ring, one also knows that $L = I\omega = \omega m r^2$.

If you are still stuck, I did assign this as a PH106 problem during the Spring 2008 semester.