Modulation techniques: low-level light measurement

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1 Hypothesis

A low-level signal can be much more effectively measured if an intentional periodic modulation is introduced.

2 Introduction

In this experiment, we would like to verify that the intensity of an LED decreases as the inverse square of the distance.[i] We need an LED and a photodetector, along with a power source or two. However, we immediately run into a problem: there is too much ambient light in the room, and worse, it is not only highly variable from hour to hour and day to day but locally influenced by things like humans walking past the experiment. What to do?

Of course we could just turn out the lights, but what if instead we figured out how to pick out *only* the light from the LED? The problem is analogous to trying to signal a distant friend with a flashlight: while they may not see the flashlight from a great distance, they will notice if you quickly turn it on and off. With our LED experiment, what we need to do then is *modulate* the intensity in a known way so we can pick the LED contribution out of the ambient light. We can do this by driving it with a sinusoidal voltage easily enough. In principle we could then put the photodetector signal on an oscilloscope to measure the signal that appears at a frequency matching the LED modulation, and we're done.

But! We can be much more clever than that, by exploiting the orthogonality of sin waves. Imagine we drive the LED at frequency ω_i, producing light with intensity modulated at that same frequency. Now we measure a signal from the photodetector. If we multiply a copy of our original reference signal with what we measure on the photodetector, we would get something like this:

$$V_{\text{ref}} = V_i \sin\left(\omega_i t +\right)$$

$$V_{\text{sig}} = V_{\text{sig}} \sin\left(\omega_{\text{sig}} t + \theta_{\text{sig}}\right)$$

$$V_{\text{ref}} \times V_{\text{sig}} = V_{\text{sig}} V_i \sin\left(\omega_i t\right) \sin\left(\omega_{\text{sig}} t + \theta_{\text{sig}}\right) = \frac{1}{2} V_{\text{sig}} V_i \cos\left[\left(\omega_i - \omega_{\text{sig}}\right) t + \theta_{\text{sig}}\right] - \frac{1}{2} V_{\text{sig}} V_i \cos\left[\left(\omega_i + \omega_{\text{sig}}\right) t + \theta_{\text{sig}}\right]$$

[i] Provided one is far enough away to treat it as a point source.

Now imagine that we have pass this resulting signal through a very sharp low-pass filter, such that $\omega_i + \omega_{\text{sig}}$ is almost completely eliminated, but $\omega_i - \omega_{\text{sig}}$ is almost completely passed. That gives us

$$V_{\text{ref}} \times V_{\text{sig}}\Big|_{\text{filtered}} = \frac{1}{2} V_{\text{sig}} V_i \cos\left[(\omega_i - \omega_{\text{sig}})\, t + \theta_{\text{sig}}\right]$$

What if ω_i and ω_{sig} are *close* but not quite matching? The product signal then has a slow oscillation at frequency $\omega_i - \omega_{\text{sig}}$. If we pass *this* signal through an integrator (which is just a low pass filter), such that we integrate over many cycles of $\omega_i - \omega_{\text{sig}}$, it will time average to zero. However, if $\omega_i = \omega_{\text{sig}}$, the product signal is constant, and the integrator will produce a constant output. If we have a perfect integrator, we'd have something like this:

$$V_{\text{ref}} \times V_{\text{sig}}\Big|_{\text{filtered \& integrated}} = \frac{1}{2} V_{\text{sig}} V_i \cos\left[\theta_{\text{sig}}\right] \delta(\omega_i - \omega_{\text{sig}})$$

Here we have used a delta function because this is still a physics class. A non-zero only if the signal and reference frequencies match, and maximized when their phase difference is zero. Why is this important? Well, pick a modulation frequency that isn't present in the ambient environment, and the odds of a spurious signal are nearly zero. (Don't choose 60 Hz, or any integer multiple of it.)

The short version: by multiplying our modulated signal and the original reference modulation, with a little signal processing, allows *phase- and frequency-sensitive detection of signals.*

3 Sketch of the Procedure

3.1 Prove the frequency selectivity

Start the HP function generator. Set it to 1 kHz with an amplitude of about 0.1 V. Connect the (sin) output to the input of the lock-in amplifier (LIA), set to "A". Turn on the LIA, and set its reference frequency to 1 kHz. Why don't you read anything? Try changing the LIA or generator frequency by a few Hz to see if it is just an offset (it isn't).

Connect the generator signal and the LIA reference output to the oscilloscope. Can you tell that the frequencies are close, but not *exactly* the same? Can you put an upper bound on the difference? (Hint: if the signals are very close in frequency, and you trigger on channel 1, channel 2 will slowly scroll by.)

Using the scope as feedback, tune the frequencies as close as you can get them, and then try plugging the generator output back into the LIA input. Any signal now? Probably not - the frequency selectivity of the lock-in is in this range probably a few mHz depending on your settings.

3.2 LED flasher

Connect the LED to the LIA output. Put the amplitude at about 2.0 V, and the frequency at 25 Hz so you can see it flash. Use a "T" connector on the LIA output to also send the signal to the oscilloscope as CH1. Once you're sure it works, bump the frequency up to a few hundred Hz, the flashing is annoying and it is easier to measure this way.

Connect the photoresistor to the hand-held multimeter. Turn the meter on and put it on the 20 kΩ resistance measuring range. We don't need to know the resistance to be honest, but this is powering the sensor. Why does this work? Using banana cables, a banana-bnc converter, and "T", connect this output to CH2 on the scope and the LIA input.

Line the photoresistor and LED on the optical rail, and gap them about 10cm apart. Can you see the modulated response from the photodetector? Cover the LED with your hand or a sheet of paper to convince yourself that you are really measuring the LED.

Move the photoresistor farther away. Even when the signal is buried in the noise and invisible on the scope, the LIA can pick it out due to the extraordinarily narrow bandwidth of the LIA measurement.

3.3 Inverse square laws

Measure the LIA intensity from the photoresistor as a function of distance d between the LED and detector. Don't get closer than about 10cm, we need the LED to approximate a point source. Measure every 10cm or so as far as you can. Does the intensity vary as $1/d^2$?

The photoresistor works by having a resistance which decreases as light intensity increases. Typically it is a power-law dependence and very nonlinear. Why does it not matter in this experiment that the photoresistor has a nonlinear response, at least in terms of the measured modulation voltage? (Hint: think about the total amount of light present vs the part you're measuring.)
Here's a sketch of what the connections should look like at this point.

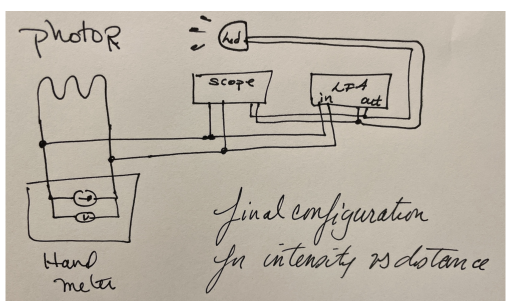

Figure 1: *The cocktail-napkin version of the final schematic.*